Vulnerable Banks[*]

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Abstract

We present a model in which fire sales propagate shocks across bank balance sheets. When a bank experiences a negative shock to its equity, a natural way to return to target leverage is to sell assets. If potential buyers are limited, then asset sales depress prices, in which case one bank's sales impact other banks with common exposures. We show how this contagion effect adds up across the banking sector, and how it can be estimated empirically using balance sheet data. We compute bank exposures to system-wide deleveraging, as well as the spillovers induced by individual banks. Applying the model to European banks, we evaluate a variety of interventions to reduce their vulnerability to fire sales during the sovereign debt crisis.

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Abstract

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I. Introduction

When a bank experiences financial stress, its troubles may spill over to other banks and threaten to contaminate the broader financial system. This is what regulators refer to when they define and measure systemic risk.

Researchers have emphasized two distinct channels by which financial shocks propagate across institutions. The first channel relies on direct linkages between banks. When two parties write a financial contract such as a swap agreement, a negative shock to one party can transmit to the other as soon as one is unable to honor the contract (e.g., Allen and Babus 2009, Gorton and Metrick 2012, Giglio, 2013). Direct linkages of this type can propagate distress, because once defaulted upon, the creditor bank may in turn lack the funds needed to deliver on its on its obligations to third parties (Duffie 2011, Kallestrup et al., 2011, Diebold and Yilmaz, 2011).

A second propagation channel involves fire-sales. When a bank sells illiquid assets to reduce its leverage, the sale may depress prices because of a lack of unconstrained buyers, which in turn can trigger financial distress at other banks that hold the same assets. Affected banks may in turn sell *other* assets in an attempt to shore up their balance sheets. Contamination can occur across seemingly unrelated assets and across seemingly unrelated institutions. Liquidation spirals of this sort have been suggested in the extensive theoretical literature on fire-sales, and are widely believed to be important drivers of systemic risk in modern financial markets.[1]

This paper develops a simple linear model of fire-sales spillovers that can be readily estimated using simple data on bank balance sheets. Our model takes as given (1) the asset holdings of each financial institution, (2) a balance-sheet adjustment rule applied by institutions when they are

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[1] See Shleifer and Vishny (1992, 2011), Kiyotaki and Moore (1997), Gromb and Vayanos (2007), Schwarcz (2008), Brunnermeier and Pedersen (2009), Allen, Babus, and Carletti (2011) for discussion of fire-sale-driven amplification.

hit by adverse shocks and (3) the liquidity of these assets on the secondary market (i.e., the price impact generated by asset liquidations). Using these assumptions, we can describe the evolution of bank balance sheets following shocks to the value of their assets.

We use the model to develop simple formulas of how fire-sale spillovers add up across banks, and how susceptible individual banks are to episodes of deleveraging by others. A key output is a measure of a bank's contribution to financial sector fragility, a quantity that we call *systemicness*. Systemicness is proportional to the product of size, leverage, and "connectedness". In our model, a bank is "connected" when it owns large and illiquid asset classes to which other banks also have high exposures. When a highly connected bank sells assets to reduce leverage, its overall impact on other banks is large, because the prices of assets it sells fall, and because the assets are held by other banks that in turn must mark down their balance sheets.

When financial regulators assess the soundness of a bank, they typically measure the vulnerability of the bank to different adverse scenarios. But our model suggests an important distinction between a bank's vulnerability and its *systemicness*. To see the distinction, consider a small but highly leveraged bank with a portfolio of risky assets. Such a bank may be quite vulnerable to financial sector deleveraging, in the sense that price impact of fire sales elsewhere in the financial system may significantly impair the bank's balance sheet. But such a bank is unlikely to be systemic, because asset sales triggered by its distress would not trigger much in the way of spillovers.

We develop a number of intuitive results regarding how the distribution of leverage and risk exposures across banks determines systemic risk. For instance, consider a negative return shock experienced by an asset that is held by a set of highly levered banks. This shock has a larger aggregate impact than if the same asset were held by less levered institutions, because the banks that hold the asset will have to sell more in a fire sale to maintain their target capital structure. More

generally, we show that the banking system is more susceptible to contagion when asset classes that are large in dollar terms are also held by the most levered banks. If the goal is to reduce fire-sale spillovers, then assets that are both volatile and illiquid should be dispersed across banks, since the same shocks generate less price impact in a deleveraging cycle. In contrast, if illiquid assets have low price volatility, then it is better to isolate these assets in separate banks, so that they are not contaminated by other assets, which in turn are subject to larger shocks.

We show how the model can be used to simulate the outcome of various policies to reduce fire-sale spillovers in the midst of a crisis. As an example of such policy analysis, consider a forced merger between two vulnerable banks—Sorkin (2009) suggests this was one of the initiatives entertained by the Federal Reserve Bank of New York during the US financial crisis. Such a policy may affect systemic risk because it redistributes existing assets across banks, which may have different exposures to shocks, different sizes, or different leverage ratios. Alternatively, consider the policy question of how to distribute a fixed amount of equity capital from the government across a large set of distressed banks. We find that from the perspective of systemic risk minimization, stabilization policies that aim to fix vulnerability at individual banks can be inferior to policies that directly target the cross-bank spillovers.

The model is straightforward to estimate using data on bank balance sheets such as is released in stress tests.[2] We apply the model to European banks during the 2010-2011 sovereign debt crisis, and use their holdings of sovereign bonds to estimate the potential spillovers that would occur in the event that a collection of European sovereigns experienced a significant haircut. Using bank holdings of sovereign bonds as inputs, we document a correlation between our estimates of bank vulnerability and equity drawdowns experienced by European banks in 2010 and 2011. We then use

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[2] See Duarte and Eisenbach (2014) for a discussion of how our model can be estimated using data from the call reports of US banks.

our data to evaluate various policy interventions. We show that forced mergers among the most exposed banks would not have reduced systemic risk very much. However, we show that modest equity injections, if distributed appropriately between the most systemic banks, can cut the vulnerability of the banking sector to deleveraging by more than half.

The remainder of the paper is organized as follows. We first develop the model, solve it, and build intuition for financial sector stability under different configurations of leverage and risk exposure across the banks. We defer our discussion of an extensive related literature to Section III, where we explain how our approach compares to other measures of systemic risk, and especially to the CoVaR and SRISK systemic risk measures developed by Adrian and Brunnermeier (2010) and Acharya, Pedersen, Philippon and Richardson (2010). In Section IV, we use commercial bank exposures provided by the EBA's July 2011 stress tests to compute the vulnerability of European banks to sovereign defaults. Section V explains how the model can be adapted to monitor vulnerability on a more dynamic basis using factor exposures. The final section concludes.

II.   A Model of Bank Deleveraging

We start by describing the assumptions. The model combines these assumptions to generate easy-to-implement measures of systemic risk.

A. Setup

There are two periods $t=1,2$, and N banks. Each bank n is financed with a mix of debt d_{nt} and equity e_{nt}. A_t is the $N \times N$ diagonal matrix of banks' assets so that each diagonal term $a_{nt} = e_{nt} + d_{nt}$ at date t. B is the $N \times N$ diagonal matrix of leverage ratios, such that each diagonal term $b_n = d_{nt}/e_{nt}$.

Each bank n holds a portfolio of K assets: m_{nk} is the weight of asset k in bank n's portfolio. M is the $N \times K$ matrix of these weights. In each period, the vector of banks' unlevered returns is:

$$R_t = MF_t, \tag{1}$$

where the $K \times 1$ vector F_t denotes asset net returns.

Assumption 1: Banks sell assets to return to leverage targets

Suppose banks receive an exogenous shock R_1 to the value of their assets at $t=1$. Because banks are levered, these shocks move banks away from their current leverage. We assume that banks respond by scaling up or down their total assets in period 2 so as to maintain a fixed leverage target on a mark-to-market basis. Such leverage-targeting is in line with empirical evidence in Adrian and Shin (2010), who show that commercial banks manage book leverage to offset shocks to asset values.

Our assumption that banks target leverage via asset sales is admittedly simplistic, but it can be easily extended to contain more realistic features. First, banks may be using equity issues to return to leverage target. Provided that equity issues and assets sales are used in fixed proportions, the analysis that follows does not change (see Appendix). Second, because loans are typically held to maturity, banks are allowed to only slowly recognize losses in their books (see for instance Milbradt, 2012). Banks can use this accounting flexibility to adjust their leverage more progressively than in our baseline model (Adrian and Shin, 2014). To account for this effect in an interesting way, the model must incorporate several rounds of deleveraging, where banks progressively get back to target leverage while slowly selling assets. We have implemented this extension, along with numerical simulations, in Appendix C. The outcome of this analysis is that progressive adjustment does not change our analysis much: The pace of asset sales slows down, but the cumulative sales are the same, and so are the induced losses. This is due to the linearity of the model.

Under the assumption of full leverage targeting, computing net asset sales is straightforward. If banks target leverage ratios given by the matrix B, then the $N{\times}1$ vector of dollar net asset increases is A_1BR_1. When $R_1{<}0$, banks with negative asset returns have to sell assets to deleverage. When $R_1{>}0$, banks with positive returns need to borrow more to preserve leverage. The intuition of this formula is simple: suppose a bank with equity of 1 and debt of 9 experiences a 10% return on its assets, bringing its equity to 2. The bank has to borrow an additional 9 and use it to buy assets to return to the prior leverage of 9-to-1.

In practice, banks will have more flexibility in dealing with a positive shock to bank equity, and so our model is most useful for thinking about dynamics following negative shocks and in which banks are highly constrained in how they return to target capital structure.

If some elements of R_1 are negative and very large, then it is possible that the A_1BR_1 vector may have some elements that are bigger in absolute value than banks' assets. This happens if the initial shock is large enough to wipe out all of the equity of the bank, in which case no amount of asset sales will return the bank to target leverage: the bank has to liquidate all of its assets, represented by the vector $A_1.(1+R_1)$. To take this situation into account, we can thus modify the vector of net asset increases by replacing it by $A_1.\max(BR_1,-1-R_1)$, where "max" is the point-wise maximum matrix operator, defined by $\max(X,Y){=}(\max(X_n,Y_n))$. In Section IV we use this modified formula, because the shocks we consider in Europe are large enough to wipe out some banks. For now we focus on the simpler linear formula.

Assumption 2: Target exposures remain fixed in percentage terms

Second, we must describe which assets are sold to return banks to target leverage. We make the simplest assumption that banks sell assets proportionately to their existing holdings, which

means that the *M* matrix remains constant between dates 1 and 2. Let ϕ be the $K \times 1$ vector of net asset (dollar) purchases by all banks in period 2. If banks keep their portfolios constant, then:

$$\phi = M' A_1 B R_1. \tag{2}$$

For example, consider a bank with holdings of 30 percent in MBS and 70 percent in loans. If the bank scales down its portfolio by ten units, it will sell 3 units of MBS and 7 units of loans (or correspondingly reduce its new lending while rolling off existing loans). Equation (2) describes this in matrix form, summed over all banks: for each bank *n* facing a shock R_{1n}, total net asset purchases are given by $a_n b_n R_{1n}$. For a given bank *n*, net purchases of asset *k* will be $m_{nk} a_n b_n R_{1n}$.

We have experimented with variations of this assumption—which is admittedly strong—to allow banks to first sell their most liquid assets (see Section IV). The constant portfolio assumption simplifies the algebra and the intuition below, but we later discuss how one can incorporate more nuanced liquidation rules that take into account different liquidity of bank assets.

Assumption 3: Fire sales generate price impact

Third, we assume that asset sales in the second period ϕ generate price impact according to a linear model:

$$F_2 = L\phi, \tag{3}$$

where *L* is a matrix of price impact ratios, expressed in units of returns per dollar of net purchase.[3] Here we implicitly assume here that *L* is diagonal, meaning that fire sales in one asset do not directly

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[3] For instance, Pulvino (1998) estimates the discount associated with fire sales of commercial aircraft by distressed airlines. In equity markets, Coval and Stafford (2007) estimate the *L* coefficient using forced purchases and sales of stock by mutual funds (see also Ellul et al, 2011, and Jotikasthira et al, 2012 who use similar methodologies in other asset markets). Bank loans can also be sold on fairly liquid markets (Drucker and Puri, 2009).

affect prices in other assets. Yet, our matrix equations below are not modified if *L* contains off-diagonal terms.[4]

Equation (3) assumes that price impact is linear, and proportional to the dollars of the asset being sold by financial institutions across the economy. This captures the spirit of most theoretical models of fire sales. To see this, suppose there are nonspecialist outside investors with a fixed dollar amount of outside wealth *W* who provide liquidity to the banking sector during a fire sale, but trade off the returns to outside projects with the returns to investing in fire sold assets. In such a setting, the equilibrium discount will be an increasing function of the total dollar amount of fire-sold assets (Shleifer and Vishny (2011), Stein (2012)).

We make the simplifying assumption that, in deciding how to return to capital structure, banks ignore the question of whether the selling is going to be productive, in the sense of being accretive to their capital. In most cases, this is a reasonable assumption, but may turn out to be more problematic when a single bank's fire-sold assets are large in dollar terms. In this case, selling the asset might be counter-productive in terms of leverage management if the price impact induced by the trade is large enough. In some extreme cases, there could be incentives to "pump" the price of a distressed asset to create the appearance of capital adequacy. In the Appendix, we describe the conditions under which asset sales are likely to be accretive to capital, in which case equation (3) is a reasonable assumption.

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[4] However, some of the intuitive expositions given below no longer hold when price impact takes on a more complex form. To the extent that off-diagonal elements are positive, this would further amplify the effects discussed below. Greenwood (2005) develops a model in which price impact spreads across similar assets. Off-diagonal price impact is reasonable when the assets being considered are similar in nature and their pricing is integrated across markets.

Combining Assumptions: Asset Returns Driven by Fire Sales

We combine equations (1), (2) and (3) to calculate the effect of bank unlevered asset returns in $t=1$ on returns in $t=2$:

$$R_2 = MF_2 = ML\phi = (MLM'BA_1)R_1. \tag{4}$$

In principle, one can iterate multiple rounds of deleveraging following an initial shock, by further multiplying by the transition matrix $MLM'BA_1$. Taken to the limit, the deleveraging process ends at a fixed point, which is a function of the eigenvalues of the transition matrix. For simplicity, we restrict our attention to the first round of deleveraging, because this delivers most of the useful intuitions about the relevant linkages between banks. In our simulations, we have experimented with continuing the deleveraging process over multiple rounds.[5]

B. Measuring Aggregate Exposures to Deleveraging ("Aggregate Vulnerability")

Consider what happens after a negative shock $-F_1 = (-f_1, ..., -f_n)$ to asset prices: this translates into dollar shocks to banks' assets given by $A_1 M F_1$. The aggregate *direct* effect on all bank assets the quantity is then $1'A_1 M F_1$, where 1 is the $N \times 1$ vector of ones. This direct effect does not involve any contagion between banks, it is simply the change in asset value driven by bank exposures. In the US and European stress tests, regulators sought to identify and quantify the direct effect across a variety of different shock scenarios.

Following equation (4), we can compute the aggregate dollar effect of shock F_1 on bank assets through fire sales. To do so, we pre-multiply $MLM'BA_1 M F_1$ by $1'A_1$. We normalize this by total bank equity pre-delevering E_1 and define "aggregate vulnerability" as:

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[5] In our empirical simulations using bank data, there is not much intuition gained by simulating the deleveraging process over additional rounds. Provided that the shocks are not too large, the rank ordering of banks in terms of their contribution to systemic risk is highly correlated across rounds.

$$AV = \frac{1'A_1MLM'BA_1MF_1}{E_1} \tag{5}$$

AV measures the percentage of aggregate bank equity that would be wiped out by bank deleveraging if there was a shock F_1 to asset returns. As a reminder, this formula omits the *direct* impact of the shock on net worth, emphasizing only the spillovers across banks. If all assets are perfectly liquid (i.e., all elements of the *L* matrix are zero), then *AV*=0: there is no contagion across banks because delevering does not involve price impact, even though there is still a direct effect of the shock on banks asset values given by $1'A_1MF_1$.

To understand the intuition behind Eq. (5), using $-R_1=-MF_1=(-r_{1t},...,-r_{nt})'$, we can rearrange terms slightly and expand:

$$AV \times E_1 = \sum_n \gamma_n b_n a_{n1} r_{n1}, \tag{6}$$

where $\gamma_n = \sum_k \left(\sum_m a_m m_{mk} \right) l_k m_{nk}$ measures the "connectedness" of bank n. This is the extent to which bank n owns large ($\sum_n a_n m_{nk}$ large) or illiquid (l_k large) asset classes. Where this is the case, one dollar of fire sales by bank n will lead to a larger amount of the banking system's holdings, since it will reduce by more the price of larger asset classes.

Equation (6) shows that the systemic risk is high when large banks (banks with large a_{n1}) are levered (large b_{n1}), exposed to the shock in question (r_{n1}), or connected (large γ_n). These properties are intuitive: if large banks are levered and/or exposed, a given shock will trigger larger asset sales. In addition, if exposed banks hold assets that are illiquid and/or widely held, then price impact is large and the overall system is more vulnerable.

More generally, the four elements of equation (6) – connectedness, leverage, size, and exposure – enter multiplicatively in determining *AV*. This means that the *distribution* of these

elements across the financial system matters enormously for systemic risk. For example, the formula tells us that from the perspective of spillovers, the cross-sectional correlation between bank size and leverage at any point in time is an important input.

C. Contribution of Individual Banks to Aggregate Deleveraging: "Systemicness"

We can calculate the contribution that each bank has -- through fire-sale spillovers -- on the aggregate vulnerability of the banking system. To do this, we again focus on the impact of a shock F_1, but assume it only affects bank n. In this case, it is easy to see that the impact coming from the liquidations of bank n on the aggregate of the banking system is:

$$S(n) = \frac{1' A_1 MLM' BA_1 \delta_n \delta'_n MF_1}{E_1},$$ (7)

where δ_n is the $N \times 1$ vector with all zeros except for the nth element, which is equal to 1. We call $S(n)$ the "systemicness" of bank n. Systemicness can be interpreted as the contribution of bank n to aggregate vulnerability, as $AV = \sum_n S(n)$.

As we did for aggregate vulnerability, we can develop intuition by expanding terms in equation (7):

$$S(n) = \gamma_n \times \left(\frac{a_n}{E_1}\right) \times b_n \times r_{n1},$$ (8)

which is the bank-level equivalent of Equation (6). Thus, a bank is more systemic if:

- *It is more connected (γ_n is bigger)*: the bank owns assets that are both illiquid and widely held by other banks.

- *It is bigger (a_n/E_1 is bigger)*: a given shock on a larger bank leads to more fire sales, which in turn leads to a large price impact.

- *It is more levered (b_n is bigger)*: a shock to a more levered bank is going to induce it to sell more, which generates more price-impact.

- *It receives a bigger shock r_{n1}* : when the magnitude (or expected volatility) of bank risk exposures increases, it can induce larger deviations from target leverage and thus large asset sales.

D. Impact of Deleveraging on each Bank: Indirect Vulnerability v. Direct Vulnerability

We define a bank's "indirect vulnerability" with respect to shock F_1 as the impact of the shock on its equity through the deleveraging of other banks:

$$IV(n) = \frac{\delta'_n A_1 MLM'BA_1 MF_1}{e_{n1}}. \tag{9}$$

$IV(n)$ measures the fraction of equity of bank n that disappears when other banks deleverage following shock F_1. It differs from *direct* vulnerability, which is the direct exposure of bank n's assets to shock F_1:

$$DV(n) = \frac{\delta'_n A_1 MF_1}{e_{n1}}. \tag{10}$$

In our empirical applications, we will systematically contrast the two measures: *IV* involves the deleveraging spiral, while *DV* does not.

To understand the intuition behind $IV(n)$, we can expand terms in equation (9):

$$IV(n) = \underbrace{(1+b_n)}_{\text{leverage}} \times \sum_k \left[\underbrace{l_k m_{nk}}_{\substack{\text{illiquidity- weighted} \\ \text{exposure to asset k}}} \times \underbrace{\left(\sum_{n'} m_{n'k} a_n b_{n'} r_{n'} \right)}_{\text{fire sales of asset k}} \right]. \tag{11}$$

The first term stands for the pure leverage effect: a given asset shock has a bigger impact on equity if the bank is more levered. The second term measures the importance of connections between banks. It is large when the bank is exposed to assets that are illiquid and exposed to heavy fire sales. Notice how $IV(n)$ differs – both conceptually and in the formula – from systemicness. Bank size enters directly into systemicness (equation (8)), but not into vulnerability (equation (11)), which is

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primarily driven by leverage and its exposure to shocks. For instance, a small bank can be vulnerable without being systemic.

E. Indirect Vulnerability of One Bank to Another

Suppose one is interested in the impact of a single bank deleveraging (for example, if it were to fail and its assets were liquidated). In this case, we can compute IV in the special case where the vector of banks' returns $R_I = \sigma \delta_m$, i.e. assuming that bank m (and only bank m) will deleverage following a shock σ to it assets. Then, following equation (9), the indirect vulnerability of bank n to a decrease of bank m's assets by $\sigma\%$ is:

$$IV(n,m) = \sigma \frac{\delta'_n A_1 MLM' BA_1 \delta_m}{e_{n1}}. \tag{12}$$

This measure captures the interdependence through deleveraging of banks n and m. $IV(n,m)$ is large when sender bank m is large and levered, when receiver bank n is levered, and more interestingly when the term $\delta_n' MLM' \delta_m$ is big, i.e., when n and m own similar illiquid assets.

F. Properties of Vulnerability and Systemicness Measures

i. Heterogeneity and Systemic Risk

One implication of equation (6) is that making the banks more similar may reduce fire sale spillovers, and thus AV. This contrasts with much of the existing literature on systemic risk, which assumes that systemic risk is high when banks have correlated stock returns.[6] The economic intuition for this result comes from two opposing effects. First, because banks liquidate all assets they hold

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[6] A notable exception is Wagner (2011) who considers a similar set of issues about the distribution of risks between banks.

when they receive shocks, shocks to liquid assets trigger fire sales of illiquid assets when banks own both types. This can make it stabilizing to ring-fence the illiquid assets into specific banks. Diversification also has an opposite effect: when all banks own all assets, any shock to asset prices will spread the fire sales across all asset holdings, which tends to reduce the total price impact. The diversification effect dominates when illiquid (high l_k) assets receive stronger shocks (high f_k). In this case, diversified (correlated) banks are better, because they can react to these shocks by partly selling liquid assets, reducing total price impact. But when liquid (low l_k) assets receive bigger shocks (high f_k), the contagion effect is more important, and stability can be increased by isolating illiquid assets into specific banks.

To illustrate this intuition more formally, consider the case of N assets and N banks of identical size a and leverage b. If assets are equally spread across banks (heterogeneity), we have $M = (11')/N$ (this is a matrix where all coefficients are equal to $1/N$) and $AV = a^2 b \sum_{n=1}^{N} \bar{l} f_n$ where $\bar{l} = (\sum_{k=1}^{N} l_k)/N$ is the average liquidity of assets. In contrast, if each asset is exclusively held by one bank dedicated to that asset (homogeneity) $M=Id.$, and $AV = a^2 b \sum_{n=1}^{N} l_n f_n$. Thus homogeneity leads to lower AV than heterogeneity if $\sum_{n=1}^{N} (\bar{l} - l_n) f_n > 0$, i.e. when assets with large shocks are more illiquid.

ii.   Equivalence of "too big to fail" and "too many to fail"

Another somewhat surprising property of AV is that it is not directly impacted by bank size. For instance, suppose we slice a bank into n smaller banks, each with the same asset mix and leverage as the original bank. It is straightforward to see from equation (6) that this leaves AV unchanged. This is because each of these new banks reacts to shocks exactly as the original bank, scaled by the ratio of their sizes. Thus, the combined impact on the rest of the system is exactly identical to that of the original bank (the aggregate quantity of each assets sold is identical). This

property is essentially a scaling property of our model, reflecting the fact that if all the assets and liabilities of a bank are increased by a constant factor, its asset sales in reaction to a shock are scaled up by that same factor. Conversely, merging banks with the same asset mix and leverage also has no impact on AV. In summary, there is no difference in our set-up between a "too-big-to-fail" problem and a "too-many-to-fail" problem induced by a series of banks similar in leverage and asset composition.

III. Relation to Literature on Systemic Risk

Our systemic risk and vulnerability measures contribute to a growing literature that studies linkages between financial institutions and the implications for fragility. Here we describe how our measures compare to prior research.

The tradition in recent papers has been to infer bank linkages from correlations in market prices. A first set of papers seeks to estimate risk directly from bond or CDS (see for instance Ang and Longstaff (2011)). Giglio (2013), for example, uses the difference between bond and CDS spreads to estimate the joint probability of failure of large banks who are sellers of protection. A second set of papers measures systemic risk through comovement in the equity returns of financial intermediaries (Adrian and Brunnermeier (2010), Acharya, Pedersen, Philippon and Richardson (2010), Billio, Getmansky, Lo, and Pelizzon (2012), Diebold and Yilmaz (2011)).

We depart from some of this literature by making simple assumptions about how funding shocks propagate across banks, i.e., we posit an economic structure to the propagation mechanism of initial shocks. To do so comes at some cost—we adopt a narrow definition of systemic risk based on banks' common exposures, thus deemphasizing bilateral risks between banks. On the other hand, the benefits are that our model-based approach can be used to do ex post policy analysis.

The structure of our model is similar to Acemoglu, Ozdaglar and Tahbaz-Salehi (2012), who study the propagation of shocks in the real economy. They derive conditions under which aggregate volatility remains high even when the network is large.

A contribution of our model relative to existing work is that it distinguishes between a bank's *contribution* to the risk of aggregate deleveraging ("systemicness"), and a bank's *sensitivity* to deleveraging by other banks ("indirect vulnerability"). Adrian and Brunnermeier (2010) define and estimate the "CoVaR" of institution n as the Value at Risk of the *whole financial sector* conditional on bank n being in distress. In our model, "systemicness" $S(n)$ is similar to their CoVaR measure; the main difference being that, while CoVaR is estimated using comovement in stock returns, we put structure on the propagation mechanism, which could result in patterns of comovement that differ from comovement of returns observed during ordinary times. On the other hand, Acharya et al. (2010) propose a measure that is closer to our "indirect vulnerability" $IV(n)$. For each bank, they estimate average returns during the 5% worst days of market conditions. They combine this estimate with bank leverage to compute the "marginal expected shortfall (MES)," which captures how much capital a bank must raise when faced with adverse market conditions. Finally, Billio, Getmansky, Lo, and Pelizzon (2012) measure systemic risk using bilateral time-series dependencies between firms. Diebold and Yilmaz (2011) discuss the relationship between cross-bank linkages estimated in this way and measures of network connectedness. Our cross-bank indirect vulnerability measure $IV(n,m)$ can provide a structural foundation for some of these connections.

Our paper is also connected to a flourishing theoretical literature that studies linkages of financial intermediaries (e.g. Eisenberg and Noe, 2001, Diebold and Yilmaz, 2011, Demange, 2011, Gouriéroux, Héam and Montfort, 2012). The main differences between these papers and ours are (1) our emphasis on deleveraging externalities (they focus on interbank contracts), and (2) the fact that

our model is easily calibratable (data on interbank lending are scarce, and often not public, and the literature is mostly theoretical). There are, however, a few important similarities. Our concept of Indirect Vulnerability ($IV(n)$) is similar in spirit to network centrality (Ballester, Calvò-Armengol and Zenou, 2006) which measures the extent to which a member of the network can easily be reached by the others. In our model, a bank is connected to others if it owns a similar, exposed, portfolio. Our concept of Systemicness ($S(n)$) is closer to the notion of "key player" (Ballester et al, 2006; Denbee, Julliard, Li and Yuan, 2012; see also the "threat index" in Demange, 2011).

Last, our analysis is closely related to policy proposals recently put forth by Duffie (2011) and Brunnermeier, Gorton, and Krishnamurthy (2011). Duffie (2011) proposes that a core group of large financial firms report their losses vis-à-vis their largest counterparties for a list of stressful scenarios. Brunnermeier, Gorton, and Krishnamurthy (2011) suggest eliciting firms' sensitivities to different risk factors and scenarios. We build on this work by modeling these sensitivities, and quantifying how these stress scenarios could play out across the financial sector.

IV. The Vulnerability of European Banks

In this Section, we apply the model to European banks in the fall of 2011. As the US subprime crisis subsided in 2009, investor attention shifted to the deteriorating fiscal position of a handful of European countries. Reflecting perceptions of default risk, interest rates on many sovereign bonds increased. Because many of these bonds were held by financial institutions, the widening interest rate spreads had the potential to cause a deleveraging cycle among the banks.

We use data on European banks' portfolios disseminated by the European Banking Authority in 2011 to estimate measures of systemicness and vulnerability that are suggested by our model. The data were released in the context of stress tests, which examined the *direct* vulnerability

of European Banks to shocks in the value of their holdings. We study the banks' susceptibility to deleveraging cycle caused by a potential writedown of sovereign bonds (although the model could in principle be used to evaluate other shocks as well). After ranking banks based on their systemicness and vulnerability, we assess quantitatively the impact of policies that were designed to contain the damage of the crisis.

A. Data

Published on the EBA website in July 2011, the European stress tests provide harmonized balance sheets for the 90 largest banks in the EU27 countries. The list includes both private and publicly listed financial companies. We draw the inputs to our calculations as follows.

Matrix A_1: The matrix of assets is obtained directly from the EBA data by taking, for each bank, the sum of all exposures. Diagonal elements a_{nn} are the "total exposure" in euros of bank n. The average exposure is €260 billion. The biggest bank is HSBC (€1440bn), the smallest one is Caixa d'Estalvis de Pollensa (€338 million).

Matrix M: To calculate the exposure matrix M, we collapse the EBA data into 42 asset classes: sovereign debt of each of the 27 EU countries plus 10 others, commercial real estate, mortgages, corporate loans, retail SME and retail revolving credit lines. The M matrix is thus a 90 x 42 matrix, where m_{nk} is the fraction of exposure to asset k of bank m. Aggregate exposure to commercial real estate across the 90 banks is €1.2 tn (5% of banking sector assets); small business lending is €744 bn (3.2%); mortgages are €4.7 tn (20%); and corporate loans are €6.7 tn (29%). Sovereign bonds account for €2.3 tn (13%).

Matrix B: The leverage matrix B is the diagonal matrix of debt-to-equity ratio. Following Adrian and Shin (2014), we use book leverage. To obtain each element b_{nn}, we divide total exposure

(the a_{nn} element of A) *minus* book equity by book equity. Because some EU banks are very levered, this number has a few outliers (leverage ratios of 540 for Allied Irish Banks, 228 for the Agricultural Bank of Greece). Because we do not want our results to be driven by these outliers, we cap target leverage b_{nn} at 30: this cap is imposed on 20 banks.

Matrix L: We assume $L=10^{-13}$ x Id, where Id is a 42 x 42 diagonal matrix of ones. We therefore assume that all 42 assets have the same price impact coefficient. 10^{-13} means that €10bn of trading imbalances lead to a price change of 10 basis points. This order of magnitude is in the neighborhood of recent empirical estimates of price impact in the bond market, such as Ellul, Jotikasthira, Lundblad, 2011, who study fire-sales of corporate bonds by insurance companies or Feldhutter, 2011, who measures selling pressure by comparing small and large trades of corporate bonds. Newman and Rierson (2003) estimate that the price impact associated with a 16-billion euro bond issuance by Deutsche Telekom was 10 basis points. Duffie, 2010, provides a summary of empirical results regarding price impact of large sales in assets markets. Nevertheless, our estimate may be an underestimate for less liquid asset classes, such as whole loans.

Shock F_1: We consider various writedowns in the value of Greek, Irish, Italian, Portugese, and Spanish debt (henceforth GIIPS debt).We study a 50% write-off of all GIIPS debt. Hence, the shock vector F_1 is equal to zero for all 42 assets, except for the five GIIPS sovereign debts, for which we assume a return of -50%. Given banks' exposures, the *direct* effect of this shock on aggregate bank equity is given by $-1'A_1MF_1$, which is equal to 381bn €, or 40.1% of aggregate bank equity. The shock we are feeding to the system is thus very large.

    B.   *Vulnerability Rankings*

We start by using the inputs above to compute vulnerability for each bank. We then validate the model by linking our vulnerability measures to the stock returns that the publicly listed banks experienced during the crisis.

To compute $IV(n)$, we use a modified version of equation (9), where we account for the fact that fire sales cannot exceed the total assets of a bank (see Section II.A.). This adjustment is necessary as some banks are so severely hit by the shock that we consider that it wipes out their equity entirely, in which case the remaining assets would be liquidated. This leads to the following definition of $IV(n)$:

$$IV(n) = \frac{\delta_n' A_1 MLM' \max(BA_1 MF_1, A_1(1 - MF_1))}{e_{n1}}$$

where $max(X,Y)$ is the element-by-element max operator. In this definition, we plug in the above matrices and the GIIPS shock vector F_1.

Table 1 lists the top 10 banks, sorted according to $IV(n)$. To see how $IV(n)$ differs from direct exposures, we also report direct vulnerability $DV(n)$, computed according to equation (10). Rankings in terms of indirect and direct effect are far from being perfectly correlated: the Spearman rank correlation between DV and IV with respect to a GIIPS shock is 0.17, and is not significantly different from zero at the 5% level. On average, the direct impact of a full-blown GIIPS crisis would be to wipe out 1.11 times the equity for the average bank. To this direct effect, the impact of the subsequent deleveraging would further wipe out some 302% of the equity of the average bank. As a reminder, all estimates of the impact of deleveraging are contingent on price impact factor L.

Between Dec 31, 2009 and July 22, 2011, European bank stocks (the subset of our sample which is publicly traded) fell by an average of 28%. What explains which banks experienced the worst returns? Surely some of the returns were explained by market participants' views about the

direct exposures that these banks had to sovereign debt. But it is also conceivable that these returns reflected indirect vulnerability $IV(n)$ to losses on GIIPS sovereign debt.

We regress cumulative returns over 2010 through July 2011 of each bank on indirect vulnerability IV, controlling for direct vulnerability DV, bank size (as measured by log of bank total exposure $log(a_{n1})$) and (target) leverage. These controls ensure that vulnerability to the deleveraging process $IV(n)$ adds explanatory power beyond a bank's direct exposure.

Table 2 shows the results of these regressions. Out of 90 banks covered by the stress tests, only 51 are publicly listed, and we have complete returns data for 49 of them. The R^2 of indirect vulnerability alone is 9%, compared with 14% when direct exposure is also included. The bank size control does not affect the estimated impact of $IV(n)$ on returns. The direct and indirect vulnerabilities have the same explanatory power on the cross-section of bank returns. For two banks that are one sample standard deviation apart in terms of $IV(n)$, cumulative returns drop by 5 percentage points more in the bank most exposed to sector-wide deleveraging. The last three columns of the table replace the dependent variable with returns computed over the shorter one-month window of July 2011 during which the stress tests were released to the public. These results are slightly weaker in magnitude, perhaps reflecting the fact that much of the information released in the stress tests was already understood by the market. Although the sample size is limited, Table 2 provides suggestive evidence that the market prices reflected indirect vulnerabilities of banks to a broader deleveraging cycle, and not only their direct exposures.

C.   *Systemicness Rankings*

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In this Section, we briefly discuss the properties of our systemicness measure $S(n)$ on European Data. As for vulnerability, we amend equations (7) and (8) to ensure that bank-level total fire sales are less than total assets:

$$S(n) = \frac{1'A_1 MLM' \delta_n \delta_n' A_1 \max(BMF_1, 1 - MF_1)}{E_n}$$

$$= \gamma_n \times \left(\frac{a_{n1}}{E_1}\right) \times \max(b_n \delta_n' MF_1, 1 - \delta_n' MF_1),$$

which shows that the systemicness of bank n can be decomposed into the product of three scalars: γ_n, which captures the impact of bank n on other banks through deleveraging, a_{nn}/E_1, which captures the relative size of bank n, and $max(b_n \delta_n' MF_1, 1 - \delta_n' MF_1)$, which reflects the size of fire sales by bank n.

Table 3 reports the systemicness ranking for the 10 most systemic banks in Europe, along with the three components of the decomposition above. Unsurprisingly, in the overall sample, systemicness is correlated with size (spearman correlation of .52, statistically significant at 1%), but this correlation is far from perfect, as can be seen among the 10 most systemic banks. For example, HSBC, the largest EU bank, does not appear in this ranking. BNP Paribas, which is the second largest, is only the fifth most systemic bank. Size is more correlated with systemicness than leverage, but size does not explain everything because there is substantial heterogeneity across banks in terms of necessary fire sales. Bankia, which is relatively small, is among the most systemic banks because fire sales would be enormous (92% of its assets), and it is highly connected with the rest of the financial system through its asset holdings (its linkage component equals 0.42). Assuming, for instance, that Bankia had an average linkage level (0.30 instead of 0.42), its systemicness would be equal to 0.29x0.95x0.30=0.08, which would make it the 8th most systemic bank instead of the 6th.

We have also examined the correlation between our measures of systemicness and SRISK from Acharya et al (2010), and Adrian and Brunnermeier's (2010) CoVar measure. SRISK is conceptually similar to our measure in that it incorporates size and leverage. For the overlapping sample of firms, SRISK is 50% correlated with Systemicness, and CoVar is 54% correlated.

The sum of systemicness across all 90 banks is equal to 2.45, which means that through the deleveraging process, our model predicts that 245% of aggregate bank equity would be wiped out (using the terminology of our model, $AV = 2.45$). This is sizeable, since the direct impact of the GIIPS writedown is only 40.1% of EU bank equity. The deleveraging effect is therefore 6 times larger than the direct shock.[7] In our following policy experiments, we focus on the deleveraging effect.

D.   Policy simulations

We now use the model to evaluate a number of different policies that have the potential to reduce deleveraging externalities. To be clear, the model does not take a position on whether banks are behaving optimally since it assumes an exogenous adjustment rule for banks. Thus, the interventions that follow should be interpreted as potential ex post interventions that could be used in a moment of crisis. The results of the experiments are reported in Table 4.

GIIPS debt re-nationalization: We start by looking at the effect of reallocating GIIPS sovereign debt to banks in their home country. This exercise is motivated by two facts. First, between July and December 2011, under pressure of markets and regulators, GIIPS-based banks increased their holdings of GIIPS debt by about 1%, while non GIIPS-based banks reduced them by

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[7] To properly calibrate this effect, we would need to amend our exercise in two directions: change the L matrix so as to account for the fact that assets are less liquid, and change the liquidation rule of banks so as to account for the fact that banks fire-sell liquid assets more. The first change would make estimates of systemic risk bigger, while the second one would tend to reduce it.

about 22%. Second, between December 2011 and January 2012, while the ECB lent about €500 bn to euro-area banks, Spanish banks bought about 23bn euro of government debt and Italian banks some €20 bn. A partially intended consequence of prudential and monetary policies over the fall of 2011 has thus been to re-nationalize GIIPS debt.

We thus implement the reallocation of 20% of aggregate holdings of each sovereign back to the balance sheets of banks of its own country. First, for each sovereign k, we aggregate euro holdings by all banks according to $s_k = \sum_n m_{nk} a_n$. For each bank n outside country k, we then remove

$$20\% \times s_k \times \frac{a_n m_{nk}}{\sum_{m \in foreign} a_m m_{mk}}$$ euro of sovereign k from its balance sheet. Then, for each domestic bank n'

in country k, we inject the holdings in proportion of its holdings of the sovereign among banks of country k: $20\% \times s_k \times \frac{a_{n'} m_{n'k}}{\sum_{m \in domestic} a_m m_{mk}}$. This reallocation never leads to negative holdings as long as

foreign banks own at least 20% of the aggregate holdings of sovereign k, which is the case in our simulation.[8]

Table 4 reports the results of this simulation. We find that it *reduces* systemic risk by about 8%. This effect is large: the amount of sovereign debt reallocated in the process is only €96 bn. What drives this surprising reduction in *AV*? As in equation (13), we can break down the overall impact into three components. Most of the effect comes through the aggregate reduction in exposure. When reallocating GIIPS debt, we are reducing GIIPS exposure of non-GIIPS banks (on average, by 0.2% of total assets), while increasing the exposure of most GIIPS banks (on average, by some 0.03% of

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[8] The only country in our sample where domestic banks own more than 80% of the aggregate bank holdings is the UK (81.6%).

their total assets).[9] Given that GIIPS banks, in our data, are on average less levered than non-GIIPS banks (with a debt-to-equity ratio of 21 compared to 23), this results in an overall reduction in AV.

Euro-bonds: Suppose we could substitute the sovereign portfolio of each bank with a new portfolio of sovereigns (1) which has the same size and (2) whose weights are the same across banks. This idea is, in effect, what policymakers had in mind when they proposed the "Eurobond." The intuition behind the Eurobond proposal was to break the loop between banks and their sovereigns, making local banks less sensitive to their own sovereign defaulting.

To implement the Eurobond idea, we change the exposure m_{nk} into $sharesov_k \times \%sov_n$ where $sharesov_k$ is the share of sovereign k in aggregate sovereign holdings, while $\%sov_n$ is the share of sovereign holdings in bank n's portfolio. This reshuffling of bonds across banks preserves each bank's total sovereign exposure, and aggregate bank holdings of each sovereign. But it makes banks more similar in terms of individual country exposure. In the context of our model, it is as if all banks were holding Eurobonds. The impact of the GIIPS shock on Eurobonds is determined by the weights of GIIPS countries vis-a-vis non-GIIPS countries in the aggregate sovereign bonds portfolio.

Table 4 shows that this policy involves a considerable reshuffling of assets across banks: some 1.6tn euro of bonds change owners. It also increases AV. As in the previous experiment, the reason is that exposure to distressed sovereigns is reallocated to banks that are more levered, so that only the "exposure change" components appears. The intuition is that non-GIIPS banks are both less exposed but more levered in the data. The eurobond experiment transfers GIIPS debt from GIIPS banks to non-GIIPS banks, and therefore increases exposure of the most levered banks.

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[9] Some GIIPS banks experience a decrease in exposure. This happens because these banks own a lot of GIIPS debt but relatively little of their own sovereign (for instance most Italian banks own much a lot of non-Italian debt, and relatively less Italian debt). As a result, the policy reduces overall exposure to GIIPS for these banks.

Ring-fencing risky assets: Perhaps more targeted policies can make the most systemic banks safer? To understand the effect of a merger, let us assume that banks indexed by *n* are merged together into a bank denoted by *. Noting that the merger preserves the quantity of assets, it is straightforward to show that (see Appendix for a formal proof):

$$\Delta AV = \sum_{m \text{ merged}} \frac{a_m}{E} \gamma_m \left(b^* r^* - b_m r_m \right). \tag{14}$$

The interpretation of equation (14) is simple: if banks that are larger or more connected have a levered exposure $b_m r_m$ lower than the merged entity $b^* r^*$, then the merger increases systemic risk. The intuition is that the merger creates contagion: banks that are relatively large and connected, but less exposed, are protected against the shock. By being merged into an entity with larger exposure, these assets become vulnerable to fire sales, increasing *AV*.

Suppose now that the regulator merges the most exposed banks into a single large bank. For each bank, we define as 'exposure' the fraction of bank equity that would be lost directly in a 50% write-down of GIIPS debt. We then study three scenarios: merge all banks with exposure above 50%, above 100% and above 150% of their own equity. This means merging 47, 20 and 9 banks, respectively.

Table 4 shows that the effect of the bank mergers is nearly zero. The reason is that the policy regroups banks that have very similar levered exposure $b_m r_m$. And, as equation (14) demonstrates, the expected change in *AV* is small when expected leverage adjusted-exposure r_{n1} is the same across merged firms. In this case, ring-fencing does not reduce systemic risk: the policy simply transforms several similar small banks into one big bank with the same exposure.

Merging exposed banks with unexposed ones: Suppose we merge the 20 most exposed banks with the banks that are unexposed to the GIIPS write-down (6 of the 90 banks do not hold any GIIPS debt). To isolate the impact of merging the two groups, we first merge the exposed banks together, then merge the unexposed banks together, and then finally perform the full merger. Merging unexposed banks does not change *AV*, because of the effect discussed in the previous experiment: they are identical with respect to the shock. For the same reason, merging exposed banks does not change things much either. Merging the two groups into one bank does, however, increase systemic risk by 20% of aggregate equity. The intuition is that the assets of unexposed banks, which were previously not sold in response to the shock, become contaminated by the poor performance of GIIPS debt.

Leverage cap: We next study the impact of capping leverage. Here, the policy is much simpler: if *x* is the cap, then, for all banks with leverage above *x*, we set $D/E = x$. We implicitly assume these banks can raise equity to reach the maximum leverage, but do not change their sizes. Economically in our model, such a policy reduces the need for banks to fire-sell assets, so it unambiguously reduces *AV*. From Equation (6) we see that:

$$\Delta AV \times E_1 = l \sum_n \left[\Delta b_n \times \underbrace{a_{n1}}_{n \text{ is large}} \times \underbrace{(-r_{n1})}_{n \text{ is large}} \times \underbrace{\left(\sum_k m_{nk} s_k \right)}_{n \text{ holds large asset classes}} \right], \text{ with } s_k = \sum_{n'} m_{n'k} a_{n'1}$$

The policy is more effective when targeted banks are either (1) bigger, (2) more exposed, or (3) hold large asset classes.

We try three different caps (knowing we capped leverage to 30 in the data): 15, 20 and 25. We calculate the amount of equity capped banks need to raise to reach this cap: for instance capping leverage at 15 (25th percentile) requires banks to raise a staggering of €480 bn. The table shows that,

to obtain a significant reduction in systemic risk, the regulator would need to set a very drastic cap. For instance, capping leverage at 25 (this is leverage at the 63rd percentile bank) only reduces vulnerability to a GIIPS shock from 245 to 238% of aggregate equity. The impact of reducing leverage to 20 is much larger.

E.   Optimizing capital injection

We have seen above that capping leverage is the only policy that delivers sizeable reductions in *AV*. The cost of this policy is large (approximately €480bn) and the action quite drastic, since it recapitalizes all banks with leverage above 15. Here we ask whether AV can be reduced by injecting different amounts of equity into different banks. The idea is to potentially identify banks are either less connected (their portfolio differs from other banks) or smaller, so that recapitalizing them may be less necessary, in which case the recapitalization can be done at lower cost.

Suppose the regulator has a given amount of cash *F* available to invest in bank equity, and cares only about reducing spillovers between banks in a deleveraging cycle. Equity injection into banks is given by the vector $f = (f_1,...,f_N)$, so that $1'f = F$. When a bank receives f_n euros of fresh equity, we assume the entire amount is used to repay existing debt, so that its debt to equity ratio becomes $(D_n - f_n)/(E_n + f_n)$.

We minimize Eq. (8) subject to the constraints that $1'f = F$ and $(D_n - f_n)/(E_n + f_n) = b_n$ for each bank. We also impose the constraint that the regulator cannot withdraw cash from equity-rich banks, so that $f_n > 0$ for all *n*.

Optimizing equity injection across banks allows us to reduce aggregate vulnerability in a more cost-effective way than any of the policy experiments we previously considered. We report in the last panel of Table 4 the reduction in *AV* obtained through the optimal injection of €200bn: *AV*

decreases by .26, which is much bigger than any other policy except the leverage cap at 15, which costs €480bn. In our model, there are large gains from optimizing.

We then seek to characterize the optimal recapitalization that comes from the model. Table 5 then reports the optimal equity injections for each bank. We only report the 20 largest banks, ranked by the size of their equity injection. This list consists mostly of Italian, Spanish and Greek banks.

By construction, the optimal injection is highly correlated with systemicness (ρ=0.91). Correlation with the four components of systemicness is lower: ρ=0.16 (leverage), ρ=0.16 (Size), ρ=0.38 (direct exposure), ρ=0.21 (linkage). This shows that when deciding to inject fresh capital into banks, the regulator should consider all components of systemicness to minimize taxpayers' investment.

F.   *Extensions and other policy interventions*

Earlier we suggested that the model could be adjusted for different liquidation rules. A natural one to consider is one in which banks first sell off their most liquid assets. Here we focus on an extreme case and show its impact on our simulations.

Suppose that banks have the flexibility to sell their sovereign bonds, but that their other assets (primarily loans) are infinitely illiquid, meaning that their early disposal would yield zero proceeds. In this case, the banks would have to concentrate their liquidations of sovereign bonds alone. In this case, we can write down a modified version of the formula for aggregate vulnerability *AV* to a shock *S*:

$$AV = \frac{1'A_{t-1}MLM^{*'}M'BA_{t-1}MS}{E_{t-1}},$$
(15)

where M^* is a weight matrix that accounts for the fact that non-sovereigns are not liquidated. Each element is given by: $m_{ik}^* = m_{ik} / \left(\sum_k m_{ik} \right)$. We only focus on factors k which corresponds to sovereign holdings. Hence, elements of M^* are bigger: banks will liquidate more sovereigns in response to an adverse shock to their balance sheets.

A striking feature of these simulations is that aggregate vulnerability is much lower. The aggregate vulnerability of banks to a GIIPS write-down is now 23%, instead of 285%. This is because changing the liquidation rule has two opposite effects. On the one hand, banks liquidate more sovereign bonds, which has a stronger price impact on other banks. On the other hand, fire sales don't contaminate other assets, which in this case are the majority of assets held on bank balance sheets. This analysis suggests that in a moment of crisis, flexibility not to sell lowers the threat of fire sales related contagion. Stepping outside of the model, one can perhaps view this as saying that flexibility not to mark all assets to market can be beneficial during times of distress.

Table 6 reports values of AV for alternative liquidation rules. We progressively add other asset classes to the list of liquid assets. As can be seen, as long as the list of liquid assets is small enough (i.e. corresponds to less than 41% of bank assets), aggregate vulnerability is reduced by illiquidity of the other assets. Illiquidity prevents banks from transmitting their shocks to otherwise immune banks. When, however, sellable assets take up a larger fraction of the balance sheet (in our simulations, this happens as soon as we include corporate loans), then the fire sale concentration effect starts dominating the "ring fencing" effect: because banks cannot liquidate everything, they sell more liquid assets, which increases the price impact and therefore contagion.

VI. Conclusions

The main purpose of stress tests is for the regulator to ensure that banks can survive economic scenarios in which the value of their assets is severely impaired. Our paper suggests that data recovered from examinations can be made useful far beyond this narrow task, and in particular can be an important input into measures of overall systemic risk. The key idea underlying our analysis is that fire sales of bank assets can spread distress across financial institutions. We use this idea, combined with assumptions about how banks will behave following shocks to net worth, to show how the resulting fire sales may spill over.

A limitation of the model is that we do not fully take into account some of the steps that banks can take to avoid selling assets. That is, our model is most useful for understanding how the dominos fall when all banks are up against hard constraints. But prior to this point, banks may have some flexibility. They may first try to sell their most liquid assets (although if all banks do this, they may discover that these assets are not so liquid). And even after running out of liquid assets to sell, banks may ask for forebearance, or use accounting flexibility to avoid recognizing losses. We have described some of these extensions here, but it is surely important to incorporate richer and more microfounded bank reaction functions into our framework.

While our model is quite stylized, it generates a number of useful insights concerning the distribution of risks in the financial sector. The model crystalizes why regulators should pay close attention to risks that are concentrated in the most levered banks. The model also suggests that microprudential policies that explicitly target bank solvency, such as was implicit in both the European and US stress tests, may be far from optimal, especially when taken from the perspective of controlling contagion.

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Table 1. Vulnerability to a 50% write-off on all GIIPS Debt. We compute the vulnerability of the major European banks to a 50% write-down on all sovereign debt of Greece, Italy, Ireland, Portugal, and Spain. In column 1, *IV(n)* denotes the indirect vulnerability via sector-wide deleveraging as we define it in Equation (10), adjusted for the fact that total fire sales are capped by total assets (see Section II.A.). In column 3, *DV(n)* denotes the direct vulnerability to the write-down on balance-sheets, as defined in Equation (9), adjusted for maximal fire sales. Both measures are normalized by bank equity. In the last line of the table, we also report sample averages: Hence, a 50% write-down on all GIIPS debt would wipe out 111% of the equity of the average bank through the direct impact, while the indirect impact via deleveraging would create an additional loss of 302% of equity.

Bank Name	Indirect Vulnerability as a Fraction of Equity		Direct Vulnerability as a Fraction of Equity	
	IV(n)	Rank	*DV(n)*	Rank
Allied Irish Banks	35.24	1	11.9	2
Agricultural Bank of Greece	12.98	2	33.5	1
West LB	8.80	3	0.9	25
Banca Monte Dei Baschi di Siena	5.08	4	3.7	3
Oesterreichische Volksbank	4.83	5	0.2	56
SNS Bank	4.71	6	0.3	55
Caixa de Aforros	4.70	7	1.4	11
NordDeustche Landesbank	4.61	8	0.4	51
Commerzbank AG	4.54	9	1.0	21
Caixa d'Estalvis de Catalunya	4.36	10	0.8	31
Full sample average	3.02		1.11	

Table 2. Vulnerability to GIIPS and Cumulative Stock Returns. For each publicly listed bank in our sample, we calculate the cumulative return between Dec 31, 1999 and July 29, 2011, or between July 1, 2011 and July 29, 2011. We then regress this return on our measure of indirect vulnerability to a sovereign debt writedown, controlling for banks' direct exposure. The writedown considered is a 50% haircut on the debt of Greece, Ireland, Italy, Portugal, and Spain. Controls include bank size (log of assets), and leverage. Robust t-statistics in brackets. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

	Cumulative Return: Dec 2009-July 2011			July 2011 Return		
	(1)	(2)	(3)	(4)	(5)	(6)
Indirect Vulnerability *IV*	-0.022***	-0.010***	-0.012***	-0.008**	-0.001	-0.002**
	[-4.52]	[-2.79]	[-3.44]	[-2.52]	[-0.94]	[-2.03]
Direct Vulnerability *DV*		-0.021***	-0.012*		-0.012***	-0.015***
		[-3.09]	[-1.94]		[-10.27]	[-4.65]
log(assets)			0.098***			-0.024
			[3.24]			[-1.01]
Debt to Equity Ratio			-0.003			0.006
			[-0.34]			[1.20]
Constant	-0.241***	-0.249***	0.282	-0.071***	-0.076***	-0.315
	[-4.25]	[-4.52]	[1.07]	[-3.12]	[-3.47]	[-1.62]
Observations	49	49	49	49	49	49
R-squared	0.094	0.150	0.248	0.070	0.181	0.235

Table 3. Systemicness Ranking in Response to a GIIPS shock. We calculate the systemicness $S(n)$ of each individual bank, assuming a 50% write-off on GIIPS sovereign debt. Systemicness is defined in equation (7) of this article and refers to the contribution of a particular bank to the aggregate vulnerability of the banking system. We report detailed information for the top ten systemic banks. Columns 2, 3, and 4 report the elements of the decomposition of systemicness from equation (8). Column 2 reports total exposure of each bank, normalized by aggregate equity. Column 3 reports the fraction of assets that would be fire-sold as a fraction of total exposure. Because of our cap, it is always smaller than 1. Column 4 focuses on the linkage effect. The last line presents the sum of systemicness over the 90 banks in our sample, which is equal to Aggregate Vulnerability AV. A 50% write-down on GIIPS debt would wipe out, through deleveraging 245% of total bank equity. Note that our decomposition accounts for the fact that fire sales induced by the write-off are capped by total assets.

Bank Name	Systemicness $S(n)$	Assets / Aggregate Banking System Equity $\left(\dfrac{a_{n1}}{E_1}\right)$	Fire sales $\max(b_n \delta'_n MF_1, 1 - \delta'_n MF_1)$	Linkage effect γ_n
Banco Santander	0.21	1.06	0.58	0.34
Unicredit	0.19	0.88	0.69	0.31
Intesa SanPaolo	0.19	0.62	0.95	0.33
BBVA	0.18	0.57	0.94	0.33
BNP Paribas	0.15	1.37	0.36	0.30
BFA-Bankia	0.12	0.29	0.95	0.42
Caja de Ahorros Y Pensiones de Barcelona	0.10	0.27	0.93	0.38
Societe Generale	0.07	0.75	0.32	0.32
Commerzbank AG	0.07	0.66	0.48	0.23
Banca Monte Dei Baschi di Siena	0.06	0.22	0.92	0.32
Full Sample Average	0.03	0.27	0.44	0.30
Full Sample Total AV	2.45			

Table 4. Impact of Various Policies on Aggregate Vulnerability of European Banking Sector. The first line reports the aggregate vulnerability of the European banks to a 50% GIIPS write-down: induced deleveraging would destroy 245% of aggregate bank equity. The remaining rows of the table show this calculation under different hypothetical policy interventions.

Policy intervention	Detail	Summary Statistics	Aggregate Vulnerability (deviation / benchmark)	Contribution of change in distribution of		
				Asset	Connectedness	Exposure
Baseline			0.00			
GIIPS debt re-nationalization (bn euros)		*Fraction of total renationalized*				
	96	0.2	-0.08	0.01	-0.01	-0.08
Eurobonds (swap individual sov. holdings		*Total amount of sovereign reshuffled (in bn €)*				
for the same basket of sovereigns)		1672	0.08	0.00	0.00	0.09
		Number of banks merged				
Merge banks on which a GIIPS shock	$x = 50\%$	47	0.13			
is at least x% of equity	$x = 100\%$	20	0.01			
	$x = 150\%$	9	0.00			
		Number of Banks Merged				
Merge banks on which a GIIPS shock	Merge exposed only	20	0.01			
is at least 100% of equity	Merge unexposed only	6	0.00			
with banks totally unexposed	Merge all	26	0.08			
		Equity Injection (in bn €)				
Leverage cap	max D/E = 15	480	-0.28			
	max D/E = 20	173	-0.11			
	max D/E = 25	45	-0.03			
Optimized equity injection of €200bn		*Countries*				
	200	All Europe	-0.26			
	200	German banks	-0.05			
	200	German + French	-0.09			
	200	GIIPS	-0.24			

Table 5. Optimal Equity Allocation to Reduce Aggregate Vulnerability to a GIIPS shock. We assume the social planner has 200bn euros to inject, and seeks the allocation of capital increases that maximizes the reduction in Aggregate Vulnerability. We only report here the top 20 receivers. Column 1 reports optimal equity injection, in billions of euros. Column 2 reports systemicness $S(n)$. Columns 3-6 provide the four components of systemicness as in equation (8) from the text: target leverage, size, exposure to the shock, and connectedness to other banks.

Bank	Equity Injection (bn euros)	Systemic ness $S(n)$	Assumed Target Leverage	Size (a_i/E_1)	Exposure to GIP shock $(e_i'MS)$	Linkage effect $(1'AMLM'e_i)$
Banca Monte Dei ...Siena	18.20	0.17	30.00	0.22	0.08	0.32
Intesa Sanpaolo S.P.A	18.20	0.23	21.43	0.62	0.05	0.33
Caja De Ahorros Y Pensiones Bcn.	17.90	0.16	22.38	0.27	0.07	0.38
Banco Bilbao Vizcaya Argentaria	17.77	0.22	20.87	0.57	0.06	0.33
Bfa-Bankia	17.40	0.16	28.63	0.29	0.05	0.42
Banco Santander S.A.	12.04	0.21	22.99	1.06	0.03	0.34
Unicredit S.P.A	12.00	0.19	22.39	0.88	0.03	0.31
Banco Popolare	8.11	0.07	30.00	0.13	0.05	0.36
Bnp Paribas	6.04	0.15	22.62	1.37	0.02	0.30
Banco De Sabadell	4.68	0.04	25.26	0.10	0.04	0.40
Banco Comercial Português	4.34	0.04	27.16	0.10	0.04	0.34
Ubi Banca	4.13	0.04	20.37	0.15	0.04	0.33
Banco Popular Español	3.53	0.03	18.50	0.14	0.04	0.35
National Bank Of Greece	3.52	0.03	12.64	0.11	0.09	0.28
Efg Eurobank Ergasias	3.26	0.03	22.88	0.08	0.06	0.26
Commerzbank Ag	3.14	0.07	30.00	0.66	0.02	0.23
Bank Of Ireland	2.98	0.03	29.36	0.17	0.02	0.32
Caja De Ahorros Del Mediterráneo	2.96	0.03	30.00	0.07	0.04	0.34
Piraeus Bank Group	2.69	0.02	16.69	0.05	0.09	0.34
Caixa De Aforros De Galicia	2.66	0.03	30.00	0.07	0.04	0.36

Table 6: Robustness to Liquidation Rules. We calculate the aggregate vulnerability *AV* to a 50% writedown of GIIPS debt. In line 1, we report the baseline AV. In line 2, we assume only sovereigns can be sold. In line 3, we assume sovereigns and commercial real estate only can be sold. In line 4, we add mortgages to the list of assets that can be sold. In line 7, we include all known assets (typically about 80 % of total exposure). The difference here with the first line is that we assume banks have no cash to adjust.

	Aggregate Vulnerability *AV*	Liquid assets / total
Benchmark *AV*	-2.85	1.00
Liquidate Sovereigns only	-0.23	0.12
+ Commercial real estate	-0.47	0.18
+ Mortgages	-2.40	0.41
+ Corporate loans	-4.11	0.68
+ Consumer loans	-4.02	0.70
+ SME loans	-3.84	0.75

Appendix A: Allowing Equity Issuance as a Substitute for Asset Sales

Below we show that if we allow banks to deleverage by raising equity, then most of the terms derived in our benchmark model (*IV*, *AV*, Systemicness) can be written in the same way but subject to a simple scaling factor.

Consider a bank with assets of 10, financed by equity of 2 and debt of 8. Suppose the bank gets a return shock that takes assets down to 9. In our model, the bank sells an additional 4 units of assets to get back to target leverage of 4-to-1. However, the bank could simply raise equity, or combine equity issuance with asset purchases. For example, if the bank raised ½ a unit of equity, and uses the proceeds to pay debt, it would only have to do asset sales of 1.5 units: indeed, its new leverage would be (1+1/2)/(8-1/2-3/2)=4, which is the target.

To describe this generally, we need to specify the propensity of each bank to rely on one tool versus the other. One simple way to do this is to assume that equity issues and asset sales both contribute at the margin to a constant fraction of deleveraging.

We proceed in two steps: we first derive a simple rule to pin-down the relative proportions of asset sales vs. equity issuance across banks. Next, following a given shock F_1, we compute the quantities of assets sold by each bank.

Step 1: A simple bank-specific rule for the relative use of asset sales vs. equity issuance in deleveraging

Consider bank n prior to any shock. Suppose that bank *n* delevers by combining the sale of a quantity q_n of assets and an issuance of equity, $s_n = x_n q_n$. At the margin, the effect of asset sales q_n and equity issuance s_n on leverage is:

$$\Delta b = \frac{\partial b_n}{\partial q_n} q_n + \frac{\partial b_n}{\partial s_n} s_n$$

Assuming that asset sales and equity issues account a constant fraction (across banks) of deleveraging means that there exists a constant x (the same for all banks) such that:

$$\frac{\left(\frac{\partial b_n}{\partial s_n} s_n\right)}{\left(\frac{\partial b_n}{\partial q_n} q_n\right)} = x$$

Now, assuming equity issuance proceeds are used to pay back debt (as asset sales), we have:

$$b_n = \frac{d_n - q_n - s_n}{a_n - d_n + s_n},$$

which implies, using simple algebra that:

$$\frac{\left(\frac{\partial b_n}{\partial s_n}\right)}{\left(\frac{\partial b_n}{\partial q_n}\right)} = 1 + \frac{d_n(a_n - d_n)}{(a_n - d_n)^2} = 1 + b_n$$

Thus, for a bank with leverage b_n, we assume that deleveraging happens in the following proportions:

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$$\frac{s_n}{q_n} = \frac{x}{1 + b_n}$$

This is just a simple deleveraging rule concerning the relative use of asset sales vs. equity issues, which we have derived from a computation at the margin of banks target leverage.

Step 2: Computing asset sales after a shock when equity is also issued to deleverage

The next step is to compute for each bank n how much equity is issued and assets sold following a shock F_1. When doing this computation, we assume the relative proportions of assets sold and debt issued are as above and that banks target to return to leverage b_n.

Applying the formulas in the paper, we have that q_n, s_n solve:

$$\frac{(d_n - q_n - s_n)}{a_n(1 + e'_n MF_1) + s_n - d_n} = b_n$$

We can then solve for q_n by rewriting this equation, using $s_n = x_n q_n$ and $b_n = d_n/(a_n - d_n)$:

$$q_i = -\frac{1}{1 + x} b_n a_n e'_n MF_1$$

The vector of net asset purchases is therefore :

$$\frac{1}{1 + x} A_1 B_1 MF_1$$

This means that our model including equity issuance can be rewritten identically to the benchmark presented in the paper, with a factor of $\frac{1}{1+x}$ in front of all terms, where x measures the intensity of the use of equity issues versus asset sales in rebalancing. In short, the results of the model do not qualitatively hinge on whether asset sales are the exclusive technique that banks use to deleverage.

Appendix B: Sequential and Partial Deleveraging

In this Appendix, we show how the model can be extended to include sequential rounds of deleveraging. We also introduce the partial leverage adjustment process.

B.1. Sequential Deleveraging Rounds

The model defined in the paper can be readily extended to multiple rounds of adjustments. The only difference is that, in this case, the asset matrix A changes every period, as banks make losses and sell assets. In other words, the asset matrix is now the state variable of the dynamic system.

Taking this into account, the system can now be written as:

$$\begin{cases} F_{t+1} = LM'A_t.\max(BMF_t, -1 - MF_t) \\ A_{t+1} = \sum_i e'_n A_t.max(0, 1 + BMF_t).(e_n e'_n) \\ IV_{t+1} = IV_t + E^{-1}.A_t MF_{t+1} \end{cases}$$

where e_n is the vector with 1 as its *n*-th component and zeros elsewhere. E^{-1} is the diagonal matrix of inverse initial equity of banks, expressed in (or euros). Banks' assets are first reduced by direct exposure to shocks, and then by any consequences from deleveraging. The *t*+1-round vulnerability vector IV_{t+1} is the vector of accumulated bank losses due to deleveraging in each round. There is no closed form because asset sizes change in each iteration.

The system evolves in one of two possible regimes. For price impact L small enough, contagion is low and IV converges geometrically. In this case, the sequential round extension does not change much the vulnerability of systemicness rankings across banks. We have verified this result using simulations and considering the same 50% GIIPS writedown shock used in this article. For price impact L large enough, however, the shocks are so big that banking assets are quickly wiped out by the rounds of deleveraging. In this case, all banks can destroy each other (systemic) and are quickly destroyed (vulnerable). In this case, the multiple round extension makes the system diverge and thus all banks are equally systemic and vulnerable.

B.2. Allowing for partial adjustment

Suppose now that banks only partially adjust their assets to restore their leverage. We model deleveraging as follows: Denote B_t^{max} the leverage matrix that would be reached by banks if they did not sell *any* assets as a response to the negative shock F_t. We will assume that, at round *t*, banks targets $\lambda B_0 + (1 - \lambda)B_t^{max}$ where λ measures the intensity by which banks seek to return to pre-shock target. The dynamics of the system can be described by:

$$
\begin{cases}
B_t^{max} = B_t \left\{ \sum_i e'_n(1 + (I + B_{t+1})MF_t).(e_n e'_n) \right\}^{-1} \\
B_{t+1} = \lambda B_0 + (1 - \lambda)B_t^{max} \\
F_{t+1} = LM'A_t.[-1 - MF_t + (I + B_{t+1})(I + B_t)^{-1} \max(0,1 + (I + B_t)MF_t)] \\
A_{t+1} = \sum_i e'_n A_t(I + B_{t+1})(I + B_t)^{-1} max(0,1 + (I + B_t)MF_t).(e_n e'_n) \\
IV_{t+1} = IV_t + E^{-1}.A_t MF_{t+1}
\end{cases}
$$

, where I is the identity matrix.

The dynamic system now has two state variables: leverage B_t and assets A_t. Every period, banks seek to return to initial leverage B_0 but only partially so.

Does slow adjustment affect the convergence of the system? One potential worry could be that banks never manage to return to target leverage B_0 because the feedback force λ is too weak, or because deleveraging spirals prevent banks from adjusting leverage enough. To check this, we have experimented with computing AV for different levels of lambda. For low values of λ, convergence is slower, but it is still achieved after only 10 rounds for $\lambda = 0.4$. But comparing the eventual levels of cumulative aggregate vulnerability, the striking fact is that they are all quite similar. With immediate adjustment ($\lambda = 1$), cumulative AV is 2.5% of aggregate equity. With very progressive adjustment ($\lambda = 0.4$), cumulative AV is 2.1%. What this simulation shows is the essentially linear nature of fire sales in our model. Whether they occur progressively or instantaneously, their cumulative impact is quite similar.

We have also investigated whether partial adjustment affects the systemicness ranking. To do this, we run simulations with varying level of price impact L, and varying adjustment-to-target-leverage speed. For low price impact, our simulations suggest that the systemicness ranking is fully preserved after 20 rounds of progressive deleveraging. For higher price impact, the ranking is less and less preserved. The intuition for this is that, when there is a lack of liquidity in the system, all assets are quickly wiped out and all banks are equally systemic.