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Short- and long-term demand curves for stocks: theory and evidence on the dynamics of arbitrage ☆

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Abstract

I develop a framework to analyze demand curves for multiple risky securities at extended horizons in a setting with limits-to-arbitrage. Following an unexpected change in uninformed investor demand for several assets, I predict returns of each security to be proportional to the contribution of that security's demand shock to the risk of a diversified arbitrage portfolio. I show that securities that are not affected by demand shocks but are correlated with securities undergoing changes in demand should experience returns related to their hedging role in arbitrageurs' portfolios. Finally, I predict a negative cross-sectional relation between post-event returns and the initial return associated with the change in demand. I confirm these predictions using data from a unique redefinition of the Nikkei 225 index in Japan, in which 255 stocks simultaneously undergo significant changes in index investor demand, causing more

than ¥2,000 billion of trading in one week and large price changes followed by subsequent reversals for all of the reweighted stocks.

1. Introduction

This paper develops a framework to analyze the dynamics of asset prices following an unexpected event in which several securities simultaneously experience varied changes in uninformed investor demand. In real-world financial markets, demand shocks frequently affect more than one security at a time, and usually in different proportions. When retail investors sell mutual fund shares, for example, their fund managers may sell a constant fraction of the fund's holdings. Simultaneous demand shocks arise in many other settings, such as index additions accompanied by deletions, index arbitrage, swap sales, and portfolio restructurings. Despite the prevalence of the phenomenon, the impact of demand shocks on large groups of securities has received little attention from financial economists, primarily because few instances exist in which exogenous cross-sectional variation in demand can be identified with any precision.

Motivated by an event in which demand shocks can be measured precisely, I develop a simple limits-to-arbitrage model that describes the path of asset prices following an unexpected simultaneous change in investor demand for a number of securities. This yields novel cross-sectional and time-series predictions, which I then test using data from the event. Consistent with previous empirical evidence, the model predicts increases in prices following an increase in demand for an asset or group of assets.[1] The change in price is proportional to the marginal contribution of the demand shock to the risk of the arbitrage portfolio. In the simple case in which demand shocks are proportional to the value-weighted market, each security has a contribution to total arbitrage risk that is proportional to beta. However, in the more general case in which demand is positive for some securities and negative for others, the model predicts that the vector of event returns is proportional to the product of the covariance of fundamental risk and the vector of demand shocks. The higher the magnitude of the demand shock, and the higher the covariance with other securities experiencing positive demand shocks, the higher is this contribution.

The second set of predictions concerns the returns of securities not directly affected by a demand shock, but which nevertheless play a hedging role in arbitrage portfolios. These stocks could become more (or less) valuable because they hedge arbitrageur positions in the affected securities. They therefore experience event

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[1]See Shleifer (1986), Harris and Gurel (1986), Goetzmann and Mark (1986), Dhillon and Johnson (1991), Lynch and Mendenhall (1997), Wurgler and Zhuravskaya (2002) and Kaul et al. (2000).

returns linked to their covariance with the securities directly affected by the demand shock. For example, stocks whose fundamentals are positively correlated with stocks experiencing positive demand shocks will, according to the theory, experience increases in price even though no direct change in demand has occurred.

The third set of predictions concerns medium- and long-run returns. I show that following a positive demand shock, prices rise initially but revert linearly over time, with the speed of reversion proportional to the initial event return, itself proportional to the marginal contribution of the demand shock to the risk of the arbitrage portfolio. Thus, the initial event return is the change in price necessary such that arbitrageurs who accommodate the demand shock have positive expected returns following the event. In the cross-section, the model predicts that post-event returns are proportional to the initial event return.

I apply the model to a unique event in which 255 securities were subjected to simultaneous uninformed demand shocks, which exceeded by far the typical daily trading volume in those securities. The event was the April 2000 redefinition of the Nikkei 225 index in Japan. As a result of the redefinition, 30 high-tech stocks replaced 30 smaller index constituents. As new index stocks represented a larger proportion of the index than the deletions, weights of the remaining 195 securities fell by nearly half. As a consequence, institutional investors tracking the Nikkei 225 index rebalanced their portfolios, buying additions and selling both the deletions and a fraction of their holdings of the 195 remainders. Total trading linked to the redefinition was about ¥2,000 billion (approximately US\$ 19 billion). During the event week, average turnover of the 255 stocks was 3.17 times the one year historical average. The additions gained an average of 19%, the deleted stocks fell by an average of 32%, and the remaining 195 stocks fell by an average of 13%. The rest of the market was nearly flat; the Tokyo Stock Exchange value-weighted index (TOPIX) dropped only 1.18% during the week. Together, the 255 stocks affected by the event represented 71% of the market capitalization on the Tokyo Stock Exchange.[2]

The event has several unique features that make it suitable for testing the model and for understanding the cross-section of short- and long-run demand curves for stock more generally. First, the redefinition involved a large number of securities, giving my study enough power to identify the model using cross-sectional variation in event and post-event returns. Second, the unusual weighting system of the Nikkei 225 yields significant variation in the size of demand shocks affecting individual stocks, which can be measured precisely. Third, the demand shocks were simultaneous, allowing me to hold other factors constant with respect to the cross-section. Fourth, concerns that index inclusion reflects economic news are less relevant in a cross-sectional study. Finally, the sheer magnitude of the event makes it well suited for studying the returns of stocks that were not directly involved in the redefinition but which could have performed a hedging role in arbitrage portfolios.

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[2]The stocks in the pre-event Nikkei index had a combined market value of ¥220 trillion on April 14, 2000. The additions had a combined market value of ¥80 trillion. Unaffected stocks on the first section of the Tokyo Stock Exchange had a combined market value of ¥125 trillion.

The model describes the short- and long-run path of prices as a function of net institutional purchases and the covariance matrix of fundamental risk of the securities. I calculate a proxy for this matrix using historical returns and find that data strongly confirm the model's predictions. Excess event returns (post-event returns) for each stock are (negatively) proportional to the marginal contribution of the demand shock to the risk of the arbitrage portfolio. I also collect data on 1,042 stocks not present in the Nikkei before or after the event. Consistent with my predictions, their returns are positively related to the change in their contribution to arbitrage risk arising because of the event. Finally, I study the reversion of returns as a function of horizon. This reveals the long-run profitability of arbitrage strategies during the event. Over 20% of the returns are reversed in the week after the event, with at least 50% more reversed during the next 20 weeks.

The results in this paper have straightforward implications for recent research that ties downward sloping demand curves to a broader range of phenomena in capital markets, such as excess volatility (Harris, 1989) and excess comovement of returns (Hardouvelis et al., 1994; Pindyck and Rotemberg, 1993; Froot and Dabora, 1999; Barberis et al., 2003), as well as for work relating variation in investor sentiment to the broad cross-section of U.S. stock returns (Baker and Wurgler, 2003). I show that demand for a stock not only influences the price of that stock, but also indirectly influences the prices of other stocks through hedging.

The paper proceeds as follows. Section 2 outlines a model of multi security arbitrage. Section 3 describes the Nikkei 225 index redefinition and presents the details of index construction and institutional rebalancing. Section 4 presents the basic tests of the model. Section 5 analyzes the profitability of different arbitrage strategies. Section 6 concludes.

2. Arbitrage with many stocks

A simple limits-to-arbitrage model describes the effects of multiple demand shocks on asset returns. The model is summarized as follows. The capital market contains many risky securities in fixed supply. On day t^*, securities receive an unexpected demand shock of varying magnitudes, changing the net supply of assets thereafter. Arbitrageurs accommodate the demand shock but receive higher expected returns in compensation for the increased risk. Expected returns linked to the event decline over time, reversing the returns incurred because of the demand shock. The framework can be readily applied to understand the effects of demand on event returns (returns between $t^* - 1$ and t^*) and the reversion of returns (returns between t^* and $t^* + k$).

To analyze both event returns and the reversion of prices after the change in net supply requires a model with many periods. I rely on a theoretical framework developed in Hong and Stein (1999) and Barberis and Shleifer (2003). Most of the derivations are left for the appendix, while the main results are presented in the text.

2.1. Setup

The capital market includes N risky securities in fixed supply given by supply vector Q. There is a risk-free asset in perfectly elastic supply with net return normalized to zero. Each security pays a liquidating dividend at some time T. The information flow regarding dividend $D_{i,T}$ is given by

$$D_{i,t} = D_{i,0} + \sum_{s=1}^{t} \varepsilon_{i,s} \quad \text{for all } i. \tag{1}$$

The information shocks $\varepsilon_{i,s}$ are announced at time s. They are identically and independently distributed over time and normal with zero mean and covariance matrix Σ. To reduce the possibility that dividends turn negative, I assume that, for all i, $D_{i,0}$ is large relative to the standard deviation of ε_i. An unfortunate characteristic of models with CARA utility and normally distributed shocks to expected dividends is that dividends can turn negative. However, the probability that this occurs is generally small as long as dividends $D_{i,0}$ are large relative to the standard deviation of ε_i. The probability can be minimized further by adding a constant growth trend in dividends. This is not modeled here for simplicity.

Two types of agents operate in the capital market. Index traders own an exogenous and fixed quantity of securities, denoted by the $N \times 1$ vector u. For now, I normalize this vector to zero. The other agents in the model are myopic mean-variance arbitrageurs. They maximize exponential utility of next period wealth subject to a wealth constraint:

$$\max_N \ E_t[-\exp(-\gamma W_{t+1})] \tag{2}$$

such that $W_{t+1} = W_t + N_t'[P_{t+1} - P_t]$.

W_t, P_t, and N_t are arbitrageurs' wealth, the vector of security prices, and arbitrageur demand at period t, respectively.

Indexers and arbitrageurs are assumed to be present in fixed mass, with no possibility for entry. This categorization is similar to the usual breakdown between informed and uninformed investors (Kyle, 1985) or between noise traders and arbitrageurs (DeLong et al., 1990). Although this assumption is not ideal in the long run, it is more innocuous at short horizons: entry into specialized arbitrage activities is not easy because of fixed costs, and moreover, a significant portion of invested money is prohibited from shorting.

I solve for the path of prices after a permanent shock to index trader demand u. Arbitrageurs form an efficient portfolio that accommodates the entire demand shock. The unconstrained solution to Eq. (2) is given by the $(N \times 1)$ demand vector

$$N_t = \frac{1}{\gamma} [Var_t(P_{t+1})]^{-1} (E_t(P_{t+1}) - P_t). \tag{3}$$

Consider the effects of a permanent demand shock. At $t = t^*$, index trader holdings increase from 0 to u. Denote positive elements of the vector u as positive demand

shocks. In equilibrium, total demand is equal to total supply

$$N_t = Q - u. \tag{4}$$

Substituting in the demand function of arbitrageurs and rearranging gives

$$P_t = E_t(P_{t+1}) - \gamma \, Var_t(P_{t+1})[Q - u]. \tag{5}$$

In forming their demands, arbitrageurs are fully rational. This means that the conditional variance of next period's prices is equal to the actual variance of next period's prices. This leads to the first proposition.

Proposition 1. *The vector of price changes following a demand shock u is given by*

$$P_{t*} - P_{t*-1} = \varepsilon_{t*} + \gamma \Sigma((T - t^*)u + Q). \tag{6}$$

The expected reversion of prices between the event period t* and the period k periods after the event is given by

$$E_{t*}(P_{t*+k} - P_{t*}) = k\gamma\Sigma(Q - u). \tag{7}$$

The covariance matrix of event price changes with the reversion of prices is given by the negative definite matrix

$$\mathrm{cov}(\Delta P_{t*}, \Delta P_{t*,t*+k}) = -(T - t^*)k\gamma^2\Sigma \cdot E(uu') \cdot \Sigma. \tag{8}$$

The diagonal terms of this matrix are all negative.

The first part of Proposition 1 states that the vector of price changes is proportional to the product of the covariance matrix of fundamentals Σ and the vector of demand shocks u, expressed as a number of shares. Intuitively, this is simply the total risk of the arbitrage portfolio. The right-hand side includes a term ΣQ, which can be interpreted as the average required return for holding the market portfolio, and ε_{t*}, the innovation in the fundamental occurring during the event week. ΣQ is proportional to the vector of CAPM betas. That is, the ith element of this vector is equal to the covariance between the value-weighted market return and the return of security i. In the absence of a shock to net supply ($u = 0$), returns are simply proportional to beta. Thus the model reduces to CAPM when there are no demand shocks. In another simple case, when the shock to supply is proportional to Q ($u = \theta Q$), event returns are again proportional to beta (see appendix).

Why does the supply shock affect prices? The two features that ensure that changes in supply affect prices are the risk aversion of arbitrageurs and the uncertainty over future fundamentals. Both are common features in rational expectations models of stock prices, such as Grossman and Stiglitz (1980).[3] If arbitrageurs were risk neutral, then the price in period t would simply be the period t expected value of the period T liquidating dividend. Should the price diverge from this expected value, arbitrageurs would take an infinitely large position against mispricing. Alternatively, if arbitrageurs were risk averse but future fundamentals were certain, the price in period t would be equal to the certain liquidating value.

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[3]Unlike their model, there is no asymmetric information here.

Again, should the price diverge from this expected value, arbitrageurs would take an infinitely large position against the mispricing, because doing so would incur no risk.

The constant of proportionality $(T - t^*)$ can be interpreted as a horizon-related multiplier. Thus, the closer the security is to liquidation, the lower is the fundamental risk faced by arbitrageurs and the lower are event returns. Following the one-time change in net supply, the only thing changing prices is innovations in fundamentals. The terminal date is important because it represents the resolution of uncertainty, and hence the end of the return reversal. As T grows large relative to t^*, the fraction of event returns reversed between any two periods falls to zero. This proportionality would not hold exactly if there were periodic shocks to index trader demand. In this case, noise would be an additional source of risk. Myopic arbitrageurs would factor in the variance of next period's prices as a result of future uncertain index trader demand. Such a model is considered for a single risky security in DeLong et al. (1990). Slezak (1994) discusses the impact of the myopia assumption on models such as these.

Focusing on the stochastic element of Eq. (6), the (excess) event return attributed to the change in net supply is given by $(T - t^*)\gamma\Sigma u$, proportional to the product of the demand shock and the covariance matrix of fundamental innovations. In the case in which the demand shock occurs in a single security, this simply says that higher arbitrage risk is associated with higher event returns, as in Wurgler and Zhuravskaya (2002). To see this, consider the $N \times 1$ vector u as a column of zeros with one positive element in position j. The event return for security j is then $(T - t^*)\gamma\sigma_j^2 u_j$, proportional to the product of the demand shock with the variance of security j.

The model provides more insight, however, in the analysis of simultaneous demand shocks to different securities. Consider again the $N \times 1$ vector u, except with a positive element u_i (corresponding to an index addition, for example) in position i and a negative element $-u_j$ (corresponding to a deletion, for example) in position j. Excess event returns are $(T - t^*)\gamma(\sigma_i^2 u_i - \rho_{ij}\sigma_i\sigma_j u_j)$ for security i and $(T - t^*)\gamma(-\sigma_j^2 u_j + \rho_{ij}\sigma_i\sigma_j u_i)$ for security j, where ρ_{ij} denotes the correlation of fundamental innovations between security i and j. If $\rho_{ij} > 0$, then the arbitrageur's negative position in stock j hedges the idiosyncratic risk incurred by the positive position in stock i. Thus, hedging reduces the magnitude of required returns in both securities. Intuitively, the ith element of the product Σu is the marginal contribution of u_i to the total risk of the arbitrage portfolio. As the number of securities affected by demand shocks increases, the more event returns for each stock are determined by the interaction of the demand shocks for other securities. Positive demand shocks thus can have negative required returns, and vice versa.

The second part of Proposition 1 concerns post-event returns. Post-event returns are negatively proportional to event returns, and reversion occurs uniformly as $t \to T$. For $T > t^*$, the reversion to fundamentals in any one of the post-event periods is smaller than the initial event return. Many studies not surprisingly have trouble detecting reversal after large demand shocks, especially if event returns are very small and innovations to fundamentals have high variance.[4]

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[4]Some studies document zero reversion (e.g., Shleifer, 1986), while others document a partial reversion

The cross-section affords more hope for detecting reversal because of the linear relation between event and post-event returns. Eq. (8) describes the covariance of the reversion of prices with changes in prices during the event. The diagonal terms of $-(T - t^*)k\gamma^2\Sigma \cdot E(uu') \cdot \Sigma$ are negative. Event returns for each stock are negatively correlated with their post-event returns.

2.2. Unaffected securities

The model also has implications for securities that are not directly affected by the demand shock. To analyze the returns of unaffected securities, consider a simplified universe with only two risky assets, 1 and 2, and the 2×1 demand vector with a positive element u_1 in position 1 and zero in position 2. Suppose that the covariance matrix of fundamentals is given by

$$\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{pmatrix}, \tag{9}$$

where ρ denotes the correlation of fundamental innovations between security 1 and 2. Applying Eq. (6), excess event returns are $(T - t^*)\gamma\sigma^2 u_1$ for security 1 and $(T - t^*)\gamma\rho\sigma^2 u_1$ for security 2. If the fundamentals of 1 and 2 are positively correlated ($\rho > 0$), then security 2 experiences positive event returns, even though index trader demand u_2 is zero. Why? The positive event returns for the addition (security 1) come from the fact that arbitrageurs must be compensated for their short position in the asset. But the risk of their short position in asset 1 is hedged by a long position in asset 2. Arbitrageurs are thus willing to expect a lower expected return in asset 2, driving up its price. In short, a positive demand shock to security 1 makes security 2 a more valuable hedge, even though index trader demand for the asset is unchanged.

To generalize for more than two risky securities requires matrix notation. Suppose the universe of assets contains $(M + N)$ securities: M securities that experience demand shocks, and N securities with zero demand shocks. The demand shock is given by the vector

$$u = \begin{pmatrix} u_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{10}$$

where u is made up of u_1, an $M \times 1$ vector of nonzero elements, followed by N zeros.

The covariance matrix of fundamentals of the $(M + N)$ risky securities can be partitioned into the $(M \times M)$ covariance matrix of fundamentals of the affected securities, the $(N \times N)$ covariance matrix of fundamentals of the unaffected securities, and the $(M \times N)$ covariance between the fundamentals of the affected

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(*footnote continued*)
(e.g., Lynch and Mendenhall (1997)) following positive demand shocks.

securities with the fundamentals of the affected securities

$$\Sigma = \begin{pmatrix} \Sigma_1 & \Phi \\ \Phi' & \Sigma_2 \end{pmatrix},$$

where

$$\Sigma_1 = M \times M,$$
$$\Sigma_2 = N \times N \tag{11}$$

and

$$\Phi = M \times N.$$

Expected event returns for the affected securities are given as before (see Proposition 1). The returns of the unaffected securities are described by Proposition 2, obtained by substituting Eqs. (10) and (11) into Eqs. (6)–(8).

Proposition 2. *Denote the* $(M \times N)$ *covariance matrix between the fundamentals of the M affected securities and the N unaffected securities by* Φ. *Then returns of securities not directly affected by the demand shock are given by*

$$P_{t^*} - P_{t^*-1} = \varepsilon_{t^*} + \gamma\Phi'((T - t^*)u_1 + Q), \tag{12}$$

where u_1 *is an* $M \times 1$ *vector of nonzero elements corresponding to the vector of demand shocks for the M affected securities. The expected reversion of prices between the event period* t^* *and the period k periods after the event is given by*

$$\mathrm{E}_{t^*}(P_{t^*+k} - P_{t^*}) = k\gamma\Phi'(Q - u_1). \tag{13}$$

2.3. Application of the model to the data

To apply Propositions 1 and 2 to the data requires a change in units from price changes to returns. This motivates Proposition 3.

Proposition 3. *Divide the universe of risky assets into M securities experiencing a demand shock (affected securities) and N securities that do not (unaffected securities). Denote the* $M \times 1$ *vector of net purchases by* ΔX, *and the* $M \times M$ *covariance matrix of fundamental returns of the M affected securities by* Σ. *Denote the* $M \times N$ *covariance matrix between the fundamental returns of the M affected securities with the N unaffected securities by* Φ.

For the affected securities, excess event returns are described by the cross-sectional regression

$$r_{it^*} = \alpha + \beta(\Sigma\Delta X)_i + \eta_{it^*} \tag{14}$$

with $\beta > 0$.

For the unaffected securities, excess event returns are described by

$$r_{it^*} = \alpha + \beta(\Phi'\Delta X)_i + \eta_{it^*},$$

Buy-and-hold post-event excess returns between period t^* and $t^* + k$, denoted R_{it^*,t^*+k}, are related to event returns by the cross-sectional regression

$$R_{it^*,t^*+k} = \alpha + \beta r_{it^*} + \eta_{ik} \tag{15}$$

with $\beta < 0$. This relation holds for both affected and unaffected securities.

Buy-and-hold post-event excess returns are related to arbitrage risk by the cross-sectional regression

$$R_{it^*,t^*+k} = \alpha + \beta(\Sigma\Delta X)_i + \eta_{ik} \tag{16}$$

for the affected securities, with $\beta < 0$. Post-event excess returns for the unaffected securities can be related to arbitrage risk by

$$R_{it^*,t^*+k} = \alpha + \beta(\Phi'\Delta X)_i + \eta_{ik} \tag{17}$$

where $\beta > 0$.

Proposition 3 merges Propositions 1 and 2 and changes the units to returns so that it can be directly applied to the event. The relation between price changes and the demand vector, expressed as a number of shares, is equivalent to the relation between returns and the demand vector expressed as price times the number of shares, or net purchases. In this paper, net purchases are always expressed in yen, and the matrix Σ denotes the covariance matrix of fundamental returns. The mapping between the units of the model and the units used in testing is derived in the appendix.

3. The Nikkei 225 redefinition

This section describes the Nikkei 225 redefinition in detail and computes the vector of demand shocks used in the cross-sectional analysis.

3.1. Event description

The Nikkei 225 is the most widely followed stock index in Japan. The newspaper *Nihon Keizai Shimbun* has maintained the index since 1970, following the discontinuation of the Tokyo Stock Exchange Adjusted Stock Price Average. The 225 index stocks are selected according to composition criteria set by the newspaper. Although index guidelines set strict targets for industry composition and liquidity requirements for individual stocks, changes to index composition have historically been infrequent. Because the structure of the index had remained relatively fixed while the industrial composition of the stock market was changing, the Nikkei had become less representative of the market over time. With the dual aim of reviving the relevance of the index and cashing in on the hype for new economy stocks, *Nihon Keizai Shimbun* announced on Friday, April 14, 2000 that rules defining index composition would change. The announcement cited changes in the "industrial and investment environment" and would become effective one week from the following
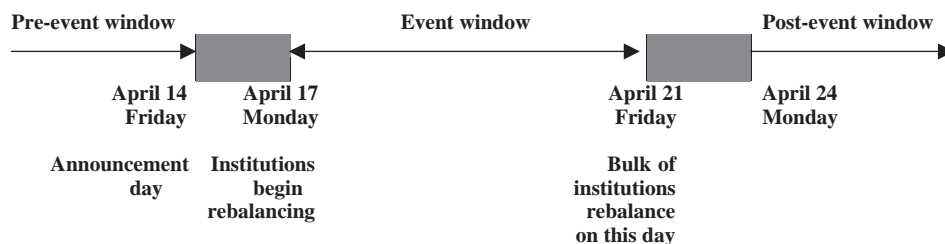
Fig. 1. Chronology. *Nihon Keizai Shimbun* announced the redefinition of the Nikkei 225 index around the close of the Tokyo Stock Exchange on Friday, April 14, 2000. The redefinition replaced 30 index securities and downweighted the remaining 195 securities. The change became effective when the market opened on April 24, 2000. To minimize tracking error, index traders would have submitted market close orders on Friday, April 21, 2000 or market open orders on Monday, April 24, 2000. Accordingly, event returns are defined on the window beginning on the close on April 14 and ending on the close on April 21. Weekly post-event windows are defined beginning on April 24.

Monday, on April 24, 2000.[5] Accordingly, for this remainder of this paper, "event window" refers to returns between April 14 and April 21, and "post-event window" refers to returns starting on April 24 (based on closing prices on April 21). This chronology is described in Fig. 1.

The index redefinition substituted 30 large capitalization stocks for 30 small capitalization stocks, downweighting by 40% the 195 stocks that remained in the index. Since the revision became effective on April 24, institutional investors tracking the Nikkei index had one full week to rebalance their portfolios. Rebalancing was complicated by the increasing prices of the additions and falling prices of the deletions during this time.

Fig. 2 plots the returns of securities affected by the Nikkei 225 index redefinition. Panel A shows the equal-weighted returns of 30 additions, 30 deletions, and 195 remainders during a one-month window surrounding the event. During the five trading days following the announcement, average returns of the additions diverged dramatically from those of the remainders and the deletions. Both remainders and deletions experienced negative returns during that week, with the average deletion falling by an extraordinary 32%. Following the event and the initial returns, the prices of additions, deletions and remainders appear to be stable. Panel B shows the same data at a longer horizon, where there appears to be substantial reversion of returns at horizons of ten to 15 weeks.

Table 1 summarizes the returns of the portfolios shown in Fig. 2. On the announcement day, April 14, returns of the additions, deletions, and remainders are all slightly negative. The news did not appear to reach the market before the close of trading. On the following Monday, the deletions fell by an average of 18.81%, while the remainders fell by 5.08% and the additions were approximately flat. The following day, the additions rose by 7.26% while the remainders and deletions

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[5]A full description of index rules can be found at http://www.nni.nikkei.co.jp/FR/SERV/ nikkei_indexes/nifaq225.html#gen1.

(A) Short-run returns (percent)

(B) Long-run returns (percent)
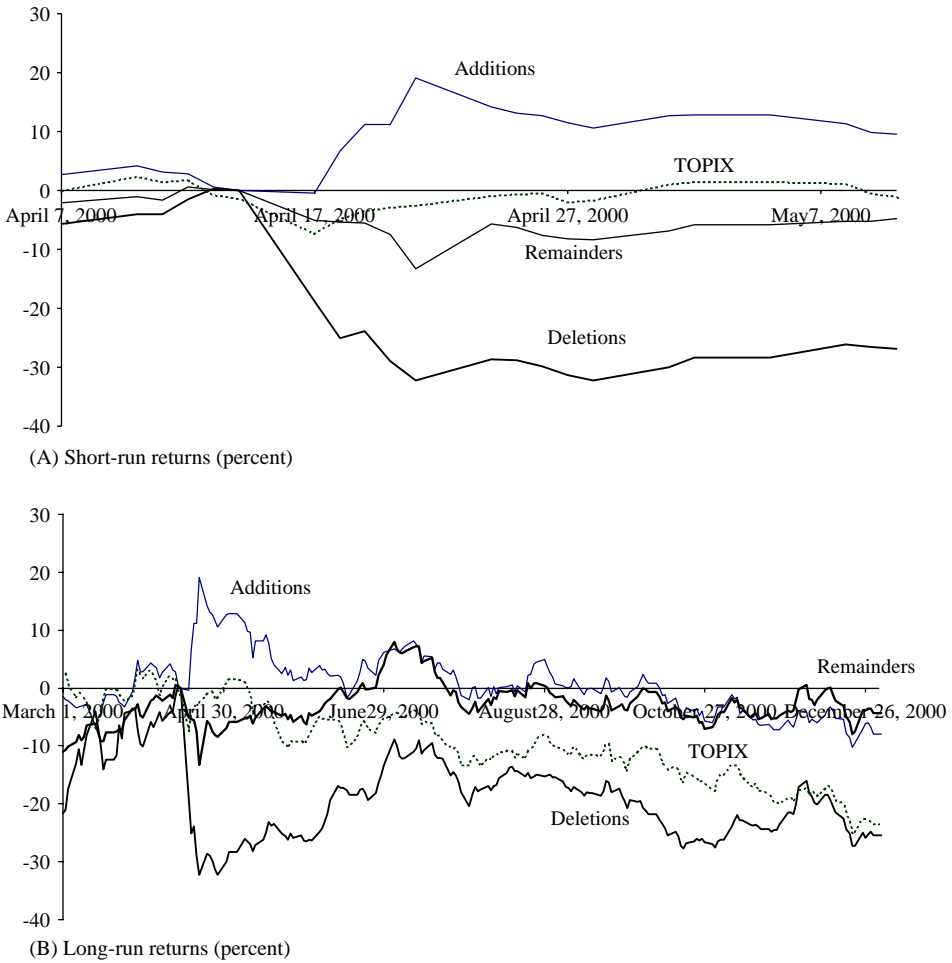
Fig. 2. Raw event returns. Buy-and-hold equal-weighted event returns for securities affected by Nikkei 225 inclusion. Additions are the 30 stocks added to the Nikkei 225 index, deletions are the 30 stocks deleted from the index, and remainders are the 195 stocks that remained in the index before and after the event. For each group of stocks, the figure plots equal-weighted buy-and-hold returns, set equal to zero on April 14, 2000. The figure also plots buy-and-hold returns of the Tokyo Stock Exchange value-weighted index (TOPIX), that is, all stocks in the first section of the Tokyo Stock Exchange. Panel (A) plots returns for a short window around the event (percent). Panel (B) shows buy-and-hold equal-weighted returns between January and December 2000 (percent).

continued to fall. By Friday of that week, additions had risen by 19.13% since the previous Friday's close, while the deletions and remainders had dropped by 32.29% and 13.35%, respectively.

Because the model has predictions for stocks not directly affected by the event, I also collect data on stocks that were not in the index before or after the redefinition. This sample is constructed as follows. I begin with approximately 1,500 stocks from

Table 1
Summary statistics, event and post-event returns

| Description | Date | Mean returns | | | | | |
		Additions	Deletions	Remainders	All	Others	TOPIX
Panel A: *Equal-weighted daily returns* (%)							
Thursday	Apr. 13	−1.99	1.78	−0.28	−0.24	−0.75	−2.48
Announcement	Apr. 14	−0.50	−0.17	−0.12	−0.17	−0.82	−0.62
Monday	Apr. 17	−0.44	−18.81	−5.08	−6.15	−5.82	−6.12
Tuesday	Apr. 18	7.26	−7.56	−0.26	−0.24	1.83	2.83
Wednesday	Apr. 19	4.21	1.79	−0.08	0.64	1.43	1.47
Thursday	Apr. 20	0.10	−6.77	−2.16	−2.43	1.76	0.57
Redefinition	Apr. 21	6.96	−4.85	−6.44	−4.67	−0.81	0.31
Monday	Apr. 24	−3.98	6.03	9.60	7.58	0.50	1.63
Tuesday	Apr. 25	−0.87	−0.32	−0.67	−0.65	0.13	0.27
Panel B: *Equal-weighted buy-and-hold weekly returns* (%)							
Event week-1	Apr. 10–14	−2.11	6.13	2.48	2.37	−0.51	−1.43
Event week	Apr. 17–21	19.13	−32.29	−13.35	−11.76	−2.08	−1.18
Event week +1	Apr. 24–28	−6.95	0.62	6.28	4.06	−0.35	0.90
Event week +2	May 1–5	2.06	6.20	2.89	3.18	3.48	3.26
Post-event five-week	Apr. 24–May 26	−13.98	9.63	9.47	6.73	2.29	−7.06
Post-event ten-week	Apr. 24–Jun. 30	−9.99	30.13	22.97	19.94	13.05	−2.60
Post-event 20-week	Apr. 24–Sep. 8	−13.80	23.09	14.01	11.81	6.59	−7.60
Post-event 40-week	Apr. 24–Jan. 26	−23.16	12.24	7.49	4.44	6.39	−20.79
N		30	30	195	255	1,042	N/A

Raw returns for securities affected by Nikkei 225 inclusion on April 24, 2000. Panel A shows average daily returns for each group of stocks starting one day before the announcement and ending two days after the close for which the rebalancing was effective. Panel B shows equal-weighted weekly returns starting two weeks prior to the event week and ending two weeks after. It also shows the buy-and-hold five-week, ten-week, 20-week, and 40-week post-event returns of a portfolio that is initially equal-weighted for each group of stocks. Post-event returns are measured relative to closing prices on April 21. Additions refer to the 30 stocks added to the index, Deletions refers to the 30 stocks deleted from the index, Remainders are the 195 stocks that remained in the index but whose weights changed. All is the 255 stocks in all groups. Others refers to a sample of 1,042 Japanese stocks not involved in the redefinition for which price and volume data were available during the event week. TOPIX is the Tokyo Stock Exchange value-weighted index, which includes all stocks traded on Section 1 of the exchange. All data are from Datastream. N/A = not applicable.

the Tokyo Stock Exchange for which Datastream collects price and volume. I exclude stocks that did not report complete data at least one year before and one year after the event. I also exclude stocks with market capitalization below that of

the smallest deletion, as these are unlikely to have been considered in any hedging strategy. The resulting sample contains 1,042 stocks. Table 1 shows average equal-weighted returns for these stocks, in addition to the returns of the value-weighted TOPIX index over the same window. The table confirms that these stocks experience similar returns to the TOPIX index during the event window.

On the date of the announcement of the index change, the combined market value of Nikkei 225 stocks was ¥225 trillion (about US$2.15 trillion), approximately 52% of the total market value on Section 1 of the Tokyo Stock Exchange. The additions had a combined market value of ¥84 trillion, and the unaffected 1,042 stocks had a combined market value of ¥184 trillion. The sum of market values of the additions, deletions, remainders, and unaffected stocks does not exactly equal the combined market value of TOPIX stocks because of data restrictions and because the sample of unaffected stocks contains some stocks that were listed on Section 2 of the Exchange. Thus the pre-event Nikkei 225 index represented roughly half of the market capitalization of TOPIX stocks, while the post-event Nikkei 225 index represented more than 70% of total market capitalization.

The second panel of Table 1 summarizes returns over longer horizons. In the week after the event, part of the initial event return was reversed. The additions fell by an average of 6.95%, the deletions gained 0.62% and the remainders gained 6.28%, about half of what they had lost during the previous week. In the ten weeks following the event, additions had an average buy-and-hold return of −9.99%, while the deletions and remainders gained 30.13% and 22.97%, respectively. These returns can be compared with the returns on the value-weighted TOPIX index, which declined by 2.60%.

Table 2 lists summary statistics for turnover during and after the event. I measure turnover for each stock as volume of shares traded divided by total shares outstanding (Lo and Wang, 2000). Prior to the event, additions had average weekly turnover of 1.02%, compared with 1.48% for the deletions and 1.16% for the remainders. During the event week, average turnover increased to 4.70% for the additions, 13.48% for the deletions, and 1.86% for the remainders. For individual stocks, turnover can be multiplied by market value to obtain yen denominated volume. Summing over the additions, deletions, and remainders, volume during the event week was ¥7.47 trillion, compared with ¥4.69 trillion the week before the event and ¥4.78 trillion the week before that. Event week volume was ¥2.5 trillion above the average volume during the previous ten weeks.

3.2. Index construction

The value of the Nikkei 225 is determined by adding the ex-rights prices ($P_{i,t}$) of its constituents, divided by the face value (FV_i), times a constant, dividing the total by the index divisor (D_t)

$$P_{\text{Nikkei},t} = \frac{\sum_{i=1}^{225} \frac{P_{i,t}}{(FV_i/50)}}{D_t}. \tag{18}$$

Table 2
Summary statistics, turnover

Description	Date	Additions	Deletions	Remainders	All	Others
Panel A: Pre-event means (%)						
Daily mean turnover		0.20	0.30	0.23	0.24	0.19
Weekly mean turnover		1.02	1.48	1.16	1.19	0.95
Panel B: Daily event turnover (%)						
Thursday	April 13	0.27	0.54	0.29	0.32	0.19
Announcement	April 14	0.27	0.82	0.35	0.40	0.18
Monday	April 17	0.83	1.68	0.34	0.55	0.21
Tuesday	April 18	0.58	2.03	0.29	0.53	0.20
Wednesday	April 19	0.65	2.12	0.29	0.55	0.16
Thursday	April 20	0.73	2.15	0.32	0.58	0.17
Redefinition at close	April 21	1.92	5.51	0.62	1.35	0.16
Monday	April 24	0.72	1.81	0.35	0.56	0.16
Tuesday	April 25	0.44	0.92	0.26	0.36	0.16
Panel C: Weekly turnover (%)						
Event week-1	April 10–14	1.14	2.38	1.34	1.44	0.95
Event week	April 17–21	4.70	13.48	1.86	3.56	0.91
Event week + 1	April 24–28	2.22	5.43	1.26	1.86	0.88
Event week + 2	May 1–5	0.45	1.06	0.38	0.47	0.48
Event week + 5 average	April 24–May 26	1.35	2.68	1.04	1.27	0.88
Event week + 10 average	April 24–June 30	1.27	2.59	1.23	1.39	0.97
Event week + 20 average	April 24–September 8	1.21	2.39	1.19	1.33	0.94
Event week + 40 average	April 24–January 26	1.20	1.84	1.10	1.20	0.84
N		30	30	195	255	1,042

Turnover for securities affected by Nikkei 225 inclusion on April 24, 2000. Turnover is defined as volume of shares traded divided by total shares outstanding. Panel A shows pre-event average equal-weighted daily and weekly turnover for the securities in each group. Panel B shows equal-weighted daily turnover starting one day before the announcement and ending two days after the close for which the rebalancing was effective. Panel C shows equal-weighted weekly turnover starting one week prior to the event week and ending two weeks after. Panel C also shows average weekly turnover for the five, ten, 20 and 40 week periods after the event. The exchange was closed Wednesday, May 3 through Friday, May 5 because of a national holiday. Turnover is correspondingly lower during this week. Additions refer to the 30 stocks added to the index. Deletions refers to the 30 stocks deleted from the index. Remainders are the 195 stocks that remained in the index but whose weights changed. All is the 255 stocks in all groups. Others refers to a sample of 1,042 Japanese stocks not involved in the redefinition for which price and volume data were available during the event week. All data are from Datastream.

Most stocks have a face value of 50, though some have face values of 5,000 or 50,000. The index divisor is adjusted daily to account for stock splits, capital changes, or stock repurchases. It is designed to preserve continuity in the index, though not necessarily in the index weights of its constituents.[6] After adjusting by face value, the index is equal-weighted in prices.

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[6]For example, following a two for one stock split of an index constituent, the effective weight of the stock falls by half, while the divisor is changed to keep the Nikkei index value unchanged.

Table 3 describes construction of the index in detail. Calculations are based on prices on April 14, with the convention that the pre-event index contains the 30 deletions and 195 remainders, and the post-event index contains the 30 additions and 195 remainders. In the subsequent analysis, net purchases are always calculated using April 14 prices, although the basic results are unchanged if prices at the end of the following week are used.[7]

Table 3 shows that out of the 30 additions, 26 have a face value of 50. They have an average price of 5,096, which results in an average weight of 1.46% in the post-event index. The last column of the table shows the Nikkei index weight divided by the weight that the stock would have taken were the index value-weighted. If the ratio is greater than one, these stocks are overweighted in the Nikkei relative to a value-weighted index; if less than one, these stocks are underweighted. For example, in a value-weighted index, the 26 additions with face value 50 would have an average weight of 0.46% (1.46/3.15). There is one addition with a face value of 5,000 and a price of 1,090,000 on April 14, 2000. This means that its price must first be divided by 100 (5,000/50) before being added to the prices of other constituents. This yields a Nikkei index weight of 3.18, which is 3.88 times the weight it would have taken in a value-weighted index. On average, the additions are overweighted in the new index. Their mean index weight is 1.42%, a factor of 2.88 greater than the hypothetical weight if the Nikkei were value-weighted.

Similar to the additions, deletions were overweighted in the old Nikkei index.[8] Although their average Nikkei weight was only 0.13%, they would have had an even lower weight in a value-weighted index. However, because additions had a larger combined weight in the index than the deletions, the total weight of the 195 remainders fell. The total weight of the remainders fell from 95.6% (0.49 × 195) to 56.6% (0.29 × 195). Effectively, this meant that institutions tracking the Nikkei index had to sell about 40% of their entire holdings simply to purchase the additions in the correct proportion.

3.3. Calculation of the vector of demand shocks

An institutional investor tracking the performance of the index would have rebalanced her portfolio with current value K to match the composition of the new

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[7] Index weights depend on prices. It follows that my estimate of net purchases depends on when I fix prices and the total funds linked to the Nikkei. During the rebalancing week, the prices of the additions rose while the prices of the deletions and remainders fell. This increased the weight of the additions in the index further still, increasing the total net purchases during this week. I use beginning-of-the-week prices to reach a conservative estimate of the size of the shock.

[8] Because I am computing equal-weighted averages, the average stock can be overweighted in the index. By definition, the value-weighted overweighting must equal one, but the equal-weighted overweighting could be greater than or less than one. I study equal-weighted averages because they are the relevant statistic in the empirical work, in which each firm is given equal weight in the statistical inference.

Table 3
Index weight changes summary statistics

| | | | | | Mean | | | |
| | | | | | Pre-event | | Post-event | |
Sample	N	Price	Price $\overline{\text{Face Value}/50}$	Nikkei 225 weight (%)	Nikkei Wgt $\overline{\text{Mkt Val Wgt}}$	Nikkei 225 weight (%)	Nikkei Wgt $\overline{\text{Mkt Val Wgt}}$
Additions							
Face value 50	26	5,096	5,096	N/A	N/A	1.46	3.15
Face value 5,000	1	1,111,000	11,100	N/A	N/A	3.18	3.88
Face value 50,000	3	1,691,667	1,691	N/A	N/A	0.48	0.25
All additions	30	210,583	4,956	N/A	N/A	1.42	2.88
Deletions							
Face value 50	30	261	261	0.13	9.14	N/A	N/A
Remainders							
Face value 50	189	960	960	0.46	2.57	0.28	2.09
Face value 500	4	4,288	429	0.21	0.42	0.12	0.34
Face value 5,000	1	1,550,000	15,500	7.45	3.85	4.44	3.14
Face value 50,000	1	1,490,000	1,490	0.72	0.07	0.43	0.06
All remainders	195	16,609	1,027	0.49	2.51	0.29	2.05

This table shows the correspondence between prices and Nikkei 225 weights of additions, deletions, and remainders and compares the index weights of these stocks to their market value as a percentage of the total market capitalization of the Nikkei 225. Stock price is the essential component in the Nikkei 225 calculation

$$P_{\text{Nikkei},t} = \frac{1}{D_t} \sum_{i=1}^{225} \frac{P_{i,t}}{F_i/50},$$

where D_t is the Nikkei 225 divisor, $P_{i,t}$ is the price of stock i on day t, and F_i is the face value of stock i ranging from 50 to 50,000. The table shows mean closing price on April 14, 2000, the day of the announcement of the Nikkei 225 redefinition. The table also reports mean stock price normalized by face value, which is the form in which prices enter the Nikkei 225 index calculation. The next two columns report the average weight in the Nikkei index and the average ratio of the Nikkei 225 weight to the weight the stock would take in the Nikkei if it were value-weighted. These two numbers are reported for both the pre-event index, which contains only the deletions and remainders, and the post-event index, which contains only the additions and remainders. All weights, including those of the additions, are based on prices on April 14, 2000. N/A = not applicable.

index. Denote the weight of security i in the index portfolio as

$$w_i = \frac{P_i/(FV_j/50)}{\frac{\sum_{j=1}^{225} \frac{P_j}{(FV_j/50)}}{D_t}}. \tag{19}$$

w_i can be interpreted as the cash value of stock i held by an investor who owns one yen worth of the index. Denoting total index capital by K, $w_i K$ is the cash amount of a stock i tied to the index.

I calculate the effects on w_i from a change in index composition under the assumption that prices remain fixed, thus yielding the size of the demand shock (expressed in yen) of each security affected by index rebalancing.

Assume that stocks $1, \ldots, M$ $(M < N)$ are replaced by $1^*, \ldots, M^*$. In the case of the Nikkei 225 rebalancing, the sum of the prices of the added securities, normalized by face value, is greater than the sum of the prices, normalized by face value, of the deleted securities:

$$\sum_{1}^{M} \frac{P_i}{(FV_i/50)} < \sum_{1^*}^{M^*} \frac{P_i^*}{(FV_i/50)}. \tag{20}$$

To calculate new index weights requires a new index divisor. The divisor is adjusted to preserve continuity in the index. This means that if the index were to close at value θ today, the new index must be defined such that it would have closed at value θ. I solve for the new index divisor $\tilde{D}_t$ as a function of the prices of the pre-event index stocks, the post-event index stocks, and the pre-event index divisor

$$\frac{\sum_{i=1}^{N} \frac{P_{i,t}}{(FV_i/50)}}{D_t} = \frac{\sum_{i=1}^{N} \frac{P_{i,t}^*}{(FV_i/50)}}{\tilde{D}_t}. \tag{21}$$

The new vector of index weights is given simply by prices, divided by face value, divided by the new index divisor. For each security i, net purchases (the demand shock) is equal to the old index weight, minus the new index weight, times the total amount of funds linked to the Nikkei 225 index. Under the assumption that index-linked assets total ¥2,430 billion (about US$ 24 billion), the sum of the absolute value of weight changes, multiplied by total assets, yields a total demand shock of ¥2.07 trillion, approximately US$ 20 billion.[9] This estimate can be considered an instrument for the actual vector of demand shocks during the event, because realized demand shocks are unobserved. Nevertheless, it is consistent with the data along two dimensions. First, event week volume was ¥2.5 trillion above the average volume for the previous ten weeks, comparable to the ¥2.07 trillion that I estimate. Second, in the cross-section, event week volume is 32% correlated with the predicted yen denominated absolute value of the demand shock for each stock.

Fig. 3 plots the distribution of net purchases of additions, deletions, and remainders, expressed in yen. The bottom panel plots the histogram of net purchases normalized by the market value of each stock. The figure reveals that, although net sales of the remainders were high when expressed in yen, they were lower than the deletions when expressed as a fraction of market capitalization. This is not surprising because the weight of the deletions in the index falls to zero, while the total weight of the remainders falls by less than half of their pre-event weights.

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[9]Total index-linked assets are quoted from Fukushima (2000). The demand shock figure corresponds to the sum of absolute values of the demand shocks for each stock. By definition, the sum of the actual values equals zero, because positive shocks by additions are offset by negative shocks by deletions and remainders.
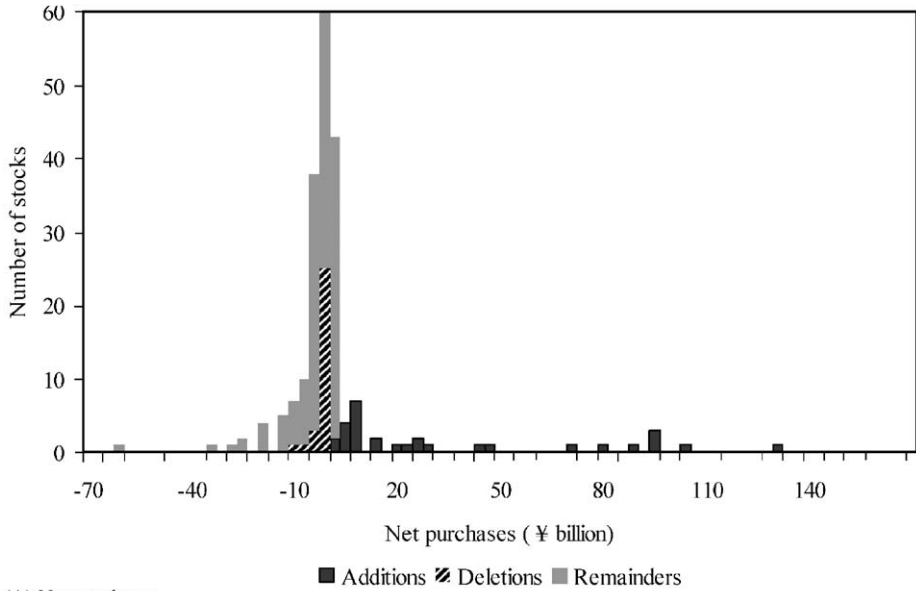
(A) Net purchases
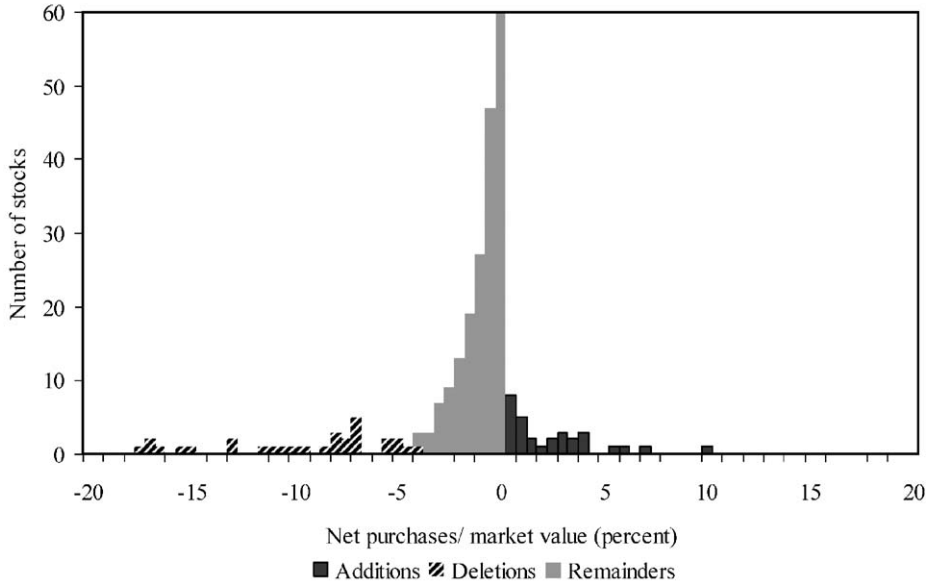
(B) Net purchases as a fraction of market value

Fig. 3. Net purchases of stocks during Nikkei 225 redefinition. Net purchases are defined as the change in index weight times estimated total institutional holdings. They are computed using implied index weights on April 14, 2000 and assuming total institutional holdings of ¥2.4 trillion. Additions, deletions, and remainders are marked separately. Panel (A) shows the distribution of net purchases, in billions of yen. Panel (B) shows the distribution of net purchases of each stock divided by that stock's market value, expressed in percentage terms.

3.4. Information content of the event

Having calculated net purchases of each stock, it is important to ask whether the change in institutional demand could have reflected new information about the future cash flows of these stocks. If this were true, event returns could partially be driven by information about fundamentals.

Generally speaking, index inclusions are an appropriate setting to study demand curves for stocks because changes in index weight provide no economic information about the future cash flows of the firms involved. There are still two questions. First, do the specific new index criteria used during the Nikkei event provide information about future cash flows? Second, if yes, how is this information correlated with the independent variables in the cross-section?

The new criteria for index selection were that components must be from the 450 most liquid stocks in the first section of the Tokyo Stock Exchange, that stocks will be divided into six sectors, and that stocks will be chosen individually after selection of sector weights.[10] The liquidity of each security was determined by looking at turnover value and rate of price change per unit of turnover. However, because criteria for inclusion and exclusion were drawn from publicly available (price and volume) historical data, the changes gave no new insight into their fundamentals. Finally, while one can debate whether index inclusion has any effect on future fundamentals, there are certainly no information effects for the 195 remainders, for which the weight change occurred only incidentally because of the difference in price between the additions and deletions. In the results that follow, a useful robustness check is to verify that all of the major findings hold on both the full sample of 255 stocks and the subsample of 195 remainders.

Index selection criteria aside, the question of whether index inclusion is informative with respect to future cash flows is less of a concern in a cross-sectional study, unless innovations in fundamentals are systematically related to the independent variables. In the Nikkei event, most of the cross-sectional variation in demand arises because the index is equal-weighted in prices. It is thus difficult to argue that cross-sectional variation of demand shocks is related to variation in news about their fundamentals.

4. The cross-section of event returns

This section presents the basic tests of the model. Following a brief discussion of estimation issues, I test the cross-sectional relationship between event returns, post-event returns, and the contribution of each security to the risk of a diversified arbitrage portfolio.

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[10]These were taken from http://www.nni.nikkei.co.jp/FR/FEAT/plunge/plunge0095.html.

4.1. Estimation issues

In most event studies, the statistical issue of greatest concern is calculating what returns would have been if the event had not occurred. Particularly in long-horizon studies, the results frequently rest on assumptions about the equilibrium rate of return.

There are two standard corrections for market movements. First, one can simply subtract the market return from event returns. Given that this paper studies a single event and the window includes the same days for all securities, subtracting the market return only changes the constant term in the cross-sectional regressions. But this means there is no way to correct for risk. Second, one can estimate pre-event betas using a market model and subtract beta times the market return. This is the technique I employ. However, a few caveats are in order. First, all of the main results hold when I run the cross-sectional regressions on raw returns, instead of risk-adjusted excess returns. Second, the market return during the week was only -1.18%. As a result, any adjustment for risk makes at most a small difference to returns. At longer horizons, the risk adjustment makes more of a difference, but the results still hold irrespective of whether I use raw or excess returns.

An important feature of my data is that the number of periods is small relative to the size of the cross-section. The incorrect assumption that estimation errors are cross-sectionally uncorrelated could yield standard errors that are biased downward by a factor of five or more (Fama and French, 2000). Many standard techniques are available to deal with this problem. I estimate the average covariance matrix of returns prior to the event and calculate ordinary least squares (OLS) standard errors under the assumption that the covariance matrix of residuals is the same during the event as in the historical data. The benefit of this procedure is that it does not depend on parameter estimates during the event. The standard errors in my cross-sectional regressions are between two and ten times the unadjusted OLS standard errors, which are not reported.

4.2. Arbitrage risk and event returns

The model suggests that event returns are linear in the contribution of each demand shock to the total risk of the arbitrageur's portfolio. This requires that I calculate the contribution of each shock to the risk of a diversified portfolio. I follow the model and multiply a proxy for the covariance matrix of fundamentals by the vector of net purchases, expressed in yen. This yields an $N \times 1$ vector, of which the ith element represents the marginal contribution of security i to total arbitrage risk. To proxy for the covariance matrix of fundamentals, I simply use the historical average covariance matrix of daily returns prior to the event, computed using three years of prior data. I have also experimented with two alternate proxies for the covariance matrix of fundamentals with similar results: the covariance matrix of excess returns and the covariance matrix of returns estimated with weekly return data. While the historical average covariance is perhaps an imperfect measure of the true risk looking forward, it likely corresponds to the technique used by real
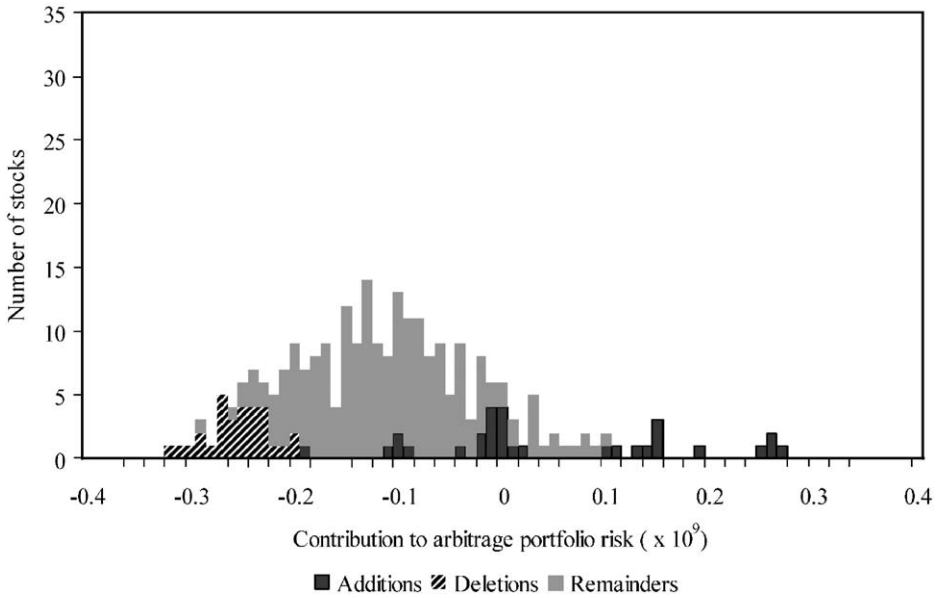
Fig. 4. Contributions to the risk of arbitrage portfolio. Histogram of contributions of each stock to the total risk of an arbitrage portfolio. For each stock, this is ith element of the product of the covariance matrix of fundamentals and the vector of yen denominated net purchases. The covariance matrix of fundamentals is estimated as the average covariance matrix of daily stock returns three years before the event. Net purchases are defined as the change in index weight times estimated total institutional holdings and are expressed in yen. They are computed using implied index weights on April 14, 2000 and assuming total institutional holdings of ¥2.4 trillion. Purchases are computed based on prices on April 14, 2000 and assuming total institutional holdings of ¥2.4 trillion. Additions, deletions, and remainders are marked separately.

arbitrageurs in determining the ex ante risk of their positions. Fig. 4 shows the distribution of this risk measure, including the 30 additions, 30 deletions, and 195 remainders. The figure reveals a high degree of cross-sectional variation. This variation appears both across the three groups of securities and within each group. Considerable overlap exists between the three groups of securities. While most of the additions have positive contributions to the total risk of the arbitrage portfolio, there are remainders with greater contributions. Most of the variation comes from the remainders.[11] Recall from the model that some remainders having positive contributions to the total risk of the arbitrage portfolio does not mean that these stocks have negative beta; they hedge the risk of some of the positions forced upon arbitrageurs by the additions and deletions.

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[11]The sum of the absolute values of the contributions to arbitrage risk is 34.36×10^9. Of this, 68.8% comes from the remainders, 8.6% from the additions and 22.7% from the deletions. The low share of the additions in this total comes from the fact that much of their idiosyncratic risk was hedged by arbitrageur short positions in the remainders and deletions.

Fig. 4 is not the same as the vector of pre-event betas for the affected securities. Only if the vector of demand shocks is proportional to the value-weighted portfolio is each stock's contribution to arbitrage risk proportional to beta.

Fig. 5 plots excess event returns against the risk measure platted in Fig. 4. Panel A shows this plot for the entire sample of affected securities, including the 30 additions, 30 deletions, and 195 remainders. The figure reveals a striking relationship between excess event returns and the contribution of each stock to the risk of the arbitrage portfolio. The additions make up most of the top right quadrant, while the deletions and remainders make up most of the bottom left quadrant. Close inspection of the figure reveals that additions, deletions, and remainders separately confirm the positive linear relationship between event returns and my risk measure.

Panel B repeats this plot, this time including the 1,042 unaffected securities. Recall from Proposition 3 that arbitrage risk for these securities is given by the demand shock weighted covariance with affected securities. I expect the returns of stocks highly correlated with the additions to be high, and the returns of stock highly correlated with the deletions and remainders to be low. For each of the unaffected stocks, I estimate the average covariance of their prior returns with each of the 30 additions, 30 deletions, and 195 remainders. This is done on the same three-year pre-event window. This yields a $255 \times 1,042$ matrix. I then multiply the transpose of this matrix by the vector of net purchases of the affected securities. The ith element of this product is the contribution of this stock to the arbitrage portfolio. Consistent with the model's predictions, excess returns for these stocks appear positively correlated with their contribution to the risk of the arbitrage portfolio. Thus stocks that tend to be positively correlated with the additions experience higher returns than stocks that tend to be positively correlated with the deletions or remainders.

Table 4 tests the relationship between arbitrage risk and event returns using the regression framework from Proposition 3. Specifically, I estimate

$$r_{it*} = a + b\Delta X_i + c(\Sigma \Delta X)_i + \eta_{it*} \tag{22}$$

on the cross-section of event returns. The independent variable ΔX_i measures the size of the demand shock, and Σ is the covariance matrix of fundamental returns. As before, the term $(\Sigma \Delta X)_i$ can be interpreted as security i's contribution to the risk of the arbitrage portfolio.

Panel A shows a strong relationship between arbitrage risk and event returns. The first line shows that, in the full sample of affected securities, stocks had large negative returns during the event week. This return is partly explained (Line 2) by the size of the demand shock. However, the coefficient on the size of the demand shock drops by half when I control for arbitrage risk (Line 3). Both the coefficient on the risk adjusted shock and the coefficient on the size of the demand shock are highly significant.

How economically significant are these results? The table shows that the R^2 on the univariate regression of event returns on the arbitrage-risk-adjusted demand shock is 0.64 and rises to 0.68 when I control for unadjusted net purchases. In short, more than half of the variation in returns during this week is explained by demand. Another measure of the economic importance of portfolio risk is the extent to which

it decreases the constant term a in the regressions. In Panel A, the significance and absolute value of the coefficient falls by about half once I control for arbitrage risk.

Panels B, C, and D repeat the baseline regressions in Panel A on the subsets of 30 additions, 30 deletions, and 195 remainders. In every case but for the additions, controlling for arbitrage risk eliminates the pure effect of the demand shock on event

Contribution to risk of arbitrage portfolio

● Deletions   ◆ Additions   − Remainders

(A) All affected securities

Contribution to risk of arbitrage portfolio

● Deletions   ◆ Additions   − Remainders   • Unaffected stocks

(B) Affected and unaffected securities
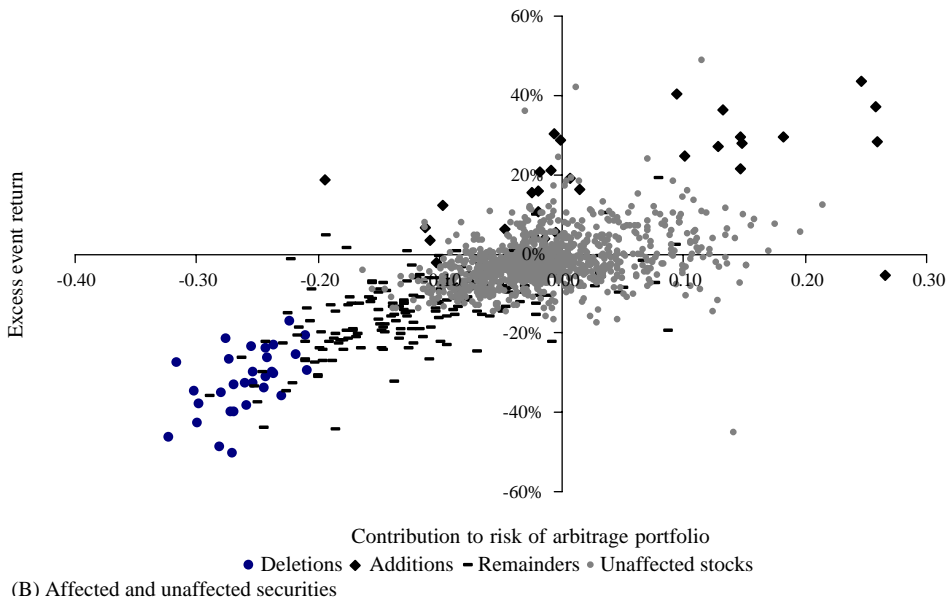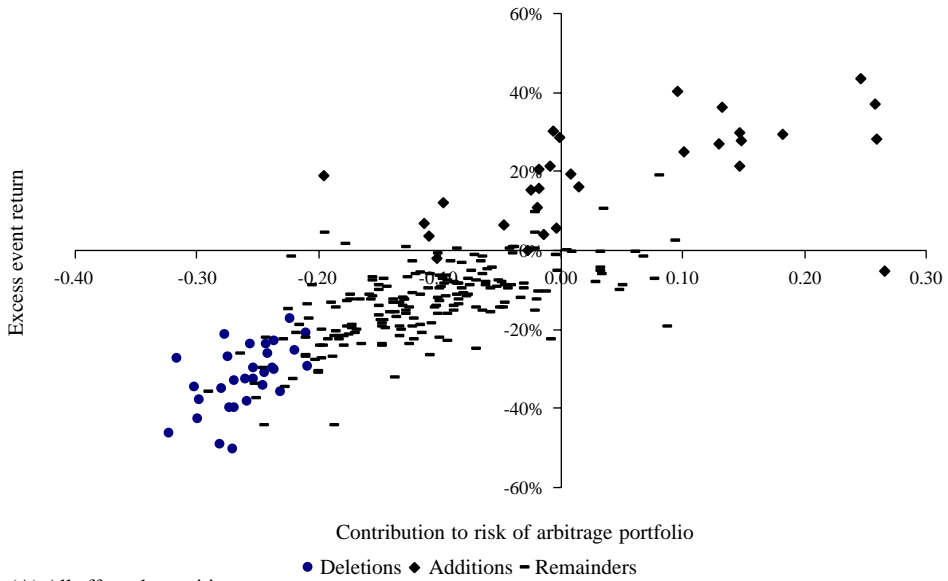
returns. The model is also successful at reducing the magnitude of the constant terms in the regression. In Panel D, for example, the model reduces the constant term from -12% to -1% by controlling for arbitrage risk.

Panel E repeats the baseline regressions from Panel A with the sample of 1,042 securities that did not experience weight changes during the event. The table shows that their contribution to arbitrage risk is significantly positively related to returns during the event week.

Panel F synthesizes the results on affected and unaffected securities by including all 1,297 stocks in the regression. In the full sample, arbitrage risk is the primary determinant of returns during this week. The R^2 of 0.50 indicates that the model explains about half of the cross-sectional variation in stock returns for all stocks in the market during the event week.

4.3. Does arbitrage portfolio risk drive returns?

It is simple to decompose the term $(\Sigma \Delta X)_i$ into its diagonal and off-diagonal components, thus separating the hedging effect from security i's diversifiable contribution to risk. Denoting the ith element of the jth row of Σ as σ_{ij} and the ith diagonal term of Σ as σ_i^2, $(\Sigma \Delta X)_i$ can be rewritten as

$$(\Sigma \Delta X)_i = \sigma_i^2 \Delta X_i + \sum_{j \neq i} \sigma_{ij} \Delta X_j. \tag{23}$$

Henceforth, let the first and second terms on the right-hand side of Eq. (23) be known as the idiosyncratic and hedging contributions to arbitrage risk, respectively. Substituting Eq. (23) into the regression model yields

$$r_{it*} = a + b\Delta X_i + c\sigma_i^2 \Delta X_i + d \sum_{j \neq i} \sigma_{ij} \Delta X_j + \eta_{it*}. \tag{24}$$

Table 5 shows estimates from this specification for each group of affected securities. The table does not consider unaffected securities because, by definition, $\sigma_i^2 \Delta X_i$ equals zero for each of these stocks.

Fig. 5. Excess event returns and arbitrage risk. Plot of excess event returns against the contribution of each stock to the total risk of an arbitrage portfolio. The vertical axis is the excess event return, defined as the raw return between April 14 and April 21, 2000 minus beta times the return on the Tokyo Stock Exchange value-weighted index (TOPIX). The horizontal axis is the contribution of each stock to the risk of the arbitrage portfolio, calculated as the ith element of the product of the covariance matrix of fundamentals and the vector of net institutional purchases. The covariance matrix of fundamentals is computed as the average covariance matrix of daily returns during the three years before the event. Net purchases are defined as the change in index weight times estimated total institutional holdings and are expressed in yen. Panel (A) includes only those securities that experienced weight changes during the event. Panel (B) includes all of the securities from Panel A plus an additional 1,042 stocks that did not experience weight changes. Additions are marked with diamonds, deletions with filled circles, remainders with dashes, and unaffected securities by dots.

Table 4
Event returns and arbitrage portfolio risk

N	a	[t]	b	[t]	c	[t]	R^2
Panel A: *All affected securities*							
255	−0.11	[−5.98]					0.00
255	−0.11	[−3.16]	0.004	[16.00]			0.27
255	0.01	[0.51]	0.002	[6.83]	1.06	[6.80]	0.68
255	0.03	[1.07]			1.20	[8.36]	0.64
Panel B: *Additions*							
30	0.20	[7.53]					0.00
30	0.13	[4.32]	0.002	[5.83]			0.44
30	0.12	[4.93]	0.003	[6.47]	−0.10	[−0.40]	0.44
30	0.18	[6.34]			0.56	[3.39]	0.28
Panel C: *Deletions*							
30	−0.31	[−5.59]					0.00
30	−0.34	[−5.92]	−0.011	[−2.51]			0.12
30	0.02	[0.25]	−0.008	[−1.70]	1.38	[3.60]	0.35
30	0.08	[0.98]			1.52	[4.15]	0.29
Panel D: *Remainders*							
195	−0.12	[−3.70]					0.00
195	−0.13	[−3.77]	−0.002	[−2.70]			0.04
195	−0.01	[−0.31]	0.002	[3.02]	0.92	[5.61]	0.45
195	−0.03	[−1.10]			0.82	[5.05]	0.43
Panel E: *Unaffected securities*							
1042	−0.01	[−0.57]					0.00
1042	−0.01	[−0.31]			0.43	[2.24]	0.13
Panel F: *All securities in sample*							
1,297	−0.03	[−1.37]					0.00
1,297	−0.03	[−1.37]	0.004	[16.00]			0.14
1,297	−0.01	[−0.26]	0.002	[9.27]	0.77	[4.85]	0.50
1,297	0.00	[−0.15]			0.84	[5.47]	0.45

Cross-sectional regressions of excess event returns on net purchases and the contribution of each stock to the risk of the arbitrage portfolio.

$$r_{it*} = a + b\Delta X_i + c(\Sigma \Delta X)_i + \eta_{it*}.$$

The dependent variable is the excess return during the event week, computed as the raw return minus beta times the return on the Tokyo Stock Exchange value-weighted index (TOPIX), where beta is estimated using a market model. The independent variables are net purchases ΔX_i, expressed in yen, and the contribution of the demand shock to the risk of a diversified arbitrage portfolio $(\Sigma \Delta X)_i$. Panel A performs the regression using data on all securities affected by the index redefinition. This covers 30 additions, 30 deletions, and 195 remainders. Panel B performs the regression using additions only. Panel C and Panel D analyze the deletions and remainders, respectively. Panel E analyzes the returns of securities that did not experience index weight changes. Because net purchases equal zero for each of these stocks, the net purchases variable is dropped from the regression. Panel F analyzes the entire sample of securities, that is, both unaffected securities and the 255 securities that experienced weight changes. Standard errors allow for cross-sectional correlation.

Table 5
A decomposition of arbitrage risk

N	a	[t]	b	[t]	c	[t]	d	[t]	R^2
Panel A: All affected securities									
255	−0.11	[−3.16]	0.004	[16.00]					0.27
255	−0.11	[−3.20]	−0.001	[−0.66]	5.22	[4.59]			0.29
255	0.02	[0.74]	0.004	[6.63]	−1.40	[−2.08]	1.11	[6.95]	0.69
255	0.01	[0.47]			2.68	[12.72]	1.06	[6.63]	0.67
Panel B: Additions									
30	0.13	[4.32]	0.002	[5.83]					0.44
30	0.13	[4.19]	0.002	[2.28]	0.05	[0.06]			0.44
30	0.12	[4.81]	0.002	[2.89]	0.01	[0.01]	−0.10	[−0.40]	0.44
30	0.14	[5.81]			2.01	[7.57]	0.02	[0.10]	0.41
Panel C: Deletions									
30	−0.34	[−5.92]	−0.011	[−2.51]					0.12
30	−0.35	[−5.84]	−0.017	[−2.04]	2.97	[0.71]			0.13
30	0.03	[0.30]	−0.006	[−0.64]	0.39	[0.09]	1.40	[3.52]	0.35
30	0.06	[0.66]			−1.80	[−0.79]	1.48	[4.01]	0.35
Panel D: Remainders									
195	−0.13	[−3.77]	−0.002	[−2.70]					0.04
195	−0.13	[−3.90]	−0.005	[−1.48]	3.23	[0.80]			0.05
195	−0.01	[−0.34]	0.003	[1.34]	−1.55	[−0.40]	0.93	[5.71]	0.45
195	−0.01	[−0.42]			3.27	[3.43]	0.90	[5.42]	0.44

Cross-sectional regressions of excess event returns on net purchases, the own variance adjusted demand shock, and the hedging contribution to portfolio risk,

$$r_{it*} = a + b\Delta X_i + c\sigma_i^2 \Delta X_i + d\sum_{j \neq i} \sigma_{ij}\Delta X_j + \eta_{it*}.$$

The independent variables are estimated net purchases of security i (ΔX_i), expressed in yen, idiosyncratic risk-adjusted net purchases ($\sigma_i^2 \Delta X_i$), and the hedging contribution of security i to arbitrage portfolio risk ($\sum_{j \neq i} \sigma_{ij}\Delta X_j$). Panel A performs the regression using data on all securities affected by the index redefinition. This covers 30 additions, 30 deletions, and 195 remainders. Panel B performs the regression using additions only. Panel C and Panel D use the remainders and deletions, respectively. Standard errors allow for cross-sectional correlation.

Panel A shows the results on the cross-section of 255 securities experiencing weight changes. The specification on the third line shows that the results in Table 4 are driven by the hedging contribution to arbitrage risk. In other words, the idiosyncratic risk of each stock is not the only determinant of the slope of the demand curve. Thus demand curves for stocks are interdependent. When I drop the raw demand shock from the regression (Line 4), the coefficients on both idiosyncratic and hedging contributions to arbitrage risk remain significant.

Panels B, C, and D repeat the basic specifications from Panel A on the additions, deletions, and remainders separately. Wherever there were significant results in

Table 4, they appear to be driven by the significance of d, the coefficient on the hedging contribution to arbitrage risk.

4.4. Robustness: controlling for liquidity

The model expresses event and post-event returns as a function of the contribution of the vector of demand shocks to the total risk of the arbitrage portfolio. I do not allow other factors, such as liquidity, to influence the slope of the demand curve for each stock. In principle, one could construct a model in which the cross-sectional variation in event returns is the result of the interaction of the demand shock for each stock and the liquidity of each stock. Such a model might specify an exogenous amount of uninformed and informed traders for each stock and derive event returns as a function of the demand shock and the share of informed traders for that stock. In the Nikkei event, thinly traded stocks might, for example, have relatively few uninformed traders and thus any trade would cause a larger price impact. The cross-sectional prediction would be that, for a demand shock of a given yen value, liquid stocks would have lower event returns.

It seems implausible that cross-sectional differences in liquidity are responsible for the variation in event returns. Most models of liquidity (e.g., Kyle, 1985) designate a single market maker for each stock, who is unable to distinguish between informed and uninformed demand. In these models, any change in demand moves prices. In the Nikkei redefinition, the quantity of demand for each stock was exogenous and common information to all market participants.

I can use the data to dismiss concerns about cross-sectional differences in liquidity entirely. First, as Table 4 reports unaffected securities experience event returns that are proportional to their contribution to the risk of the arbitrage portfolio. In a liquidity-based model, these securities do not experience demand shocks and thus should not experience event returns. The result can be reconciled only with a model in which the unaffected securities play a role in hedging the risk incurred because of arbitrageur positions in the other securities.

Second, I reestimate the baseline regression of returns on the contribution to arbitrage risk. I now include measures of demand shock scaled by proxies for liquidity on the right-hand side:

$$r_{it*} = a + b\Delta X_i + c(\Sigma\Delta X)_i + d(\Delta X_i/\overline{Vol_i}) + e(\Delta X_i/MV_i) + \eta_{it*}. \qquad (25)$$

As before, the coefficient c measures the sensitivity of returns to the contribution of the stock to the risk of an arbitrage portfolio. The next term on the right-hand side $(\Delta X_i/\overline{Vol_i})$, is net purchases divided by average trading volume. The last term $(\Delta X_i/MV_i)$, is net purchases divided by market capitalization. Both of these can be thought of as stock specific normalizations of demand shocks. Because net purchases (ΔX_i) are zero for all unaffected stocks, I estimate these regressions only on the sample of stocks that experienced weight changes during the Nikkei redefinition.

Table 6 shows these results. Panel A shows estimates for the entire sample of affected securities. The coefficient on the contribution to arbitrage risk increases from 1.06 to 1.19 once I add the control for liquidity, as proxied by trading volume.

Table 6
Event returns and arbitrage risk including various liquidity controls

N	a	[t]	b	[t]	c	[t]	d	[t]	e	[t]	R^2
Panel A: All affected securities											
255	0.01	[0.51]	0.002	[6.83]	1.06	[6.80]					0.68
255	0.03	[1.08]			1.19	[8.31]	0.00	[1.83]			0.64
255	0.02	[0.73]			0.95	[6.71]			1.07	[4.62]	0.67
255	0.02	[0.74]			0.95	[6.70]	0.00	[1.13]	1.06	[4.57]	0.67
255	0.01	[0.20]	0.002	[6.40]	0.83	[5.50]	0.00	[0.89]	1.02	[4.30]	0.71
Panel B: Additions											
30	0.12	[4.93]	0.003	[6.47]	−0.10	[−0.40]					0.44
30	0.10	[2.75]			0.28	[1.58]	0.01	[5.46]			0.47
30	0.14	[4.65]			0.33	[1.80]			2.19	[4.11]	0.38
30	0.09	[2.65]			0.29	[1.60]	0.01	[3.88]	−0.45	[−0.53]	0.47
30	0.07	[2.09]	0.002	[4.25]	−0.15	[−0.62]	0.01	[2.93]	−0.40	[−0.47]	0.56
Panel C: Deletions											
30	0.02	[0.25]	−0.008	[−1.70]	1.38	[3.60]					0.35
30	0.03	[0.34]			0.93	[2.30]	0.00	[3.03]			0.41
30	0.09	[1.02]			1.42	[3.81]			0.31	[1.30]	0.32
30	0.03	[0.40]			0.91	[2.24]	0.00	[2.81]	0.16	[0.65]	0.41
30	−0.07	[−0.73]	−0.016	[−2.95]	0.51	[1.15]	0.00	[2.46]	0.75	[2.89]	0.55
Panel D: Remainders											
195	−0.01	[−0.31]	0.002	[3.02]	0.92	[5.61]					0.45
195	−0.03	[−1.10]			0.82	[5.05]	0.00	[−0.02]			0.43
195	−0.02	[−0.91]			0.71	[4.35]			1.56	[1.70]	0.44
195	−0.02	[−0.91]			0.71	[4.34]	0.00	[0.25]	1.57	[1.71]	0.44
195	−0.01	[−0.33]	0.002	[2.78]	0.82	[5.16]	0.00	[0.07]	1.17	[1.31]	0.45
Panel E: All securities in sample											
1,297	−0.01	[−0.26]	0.002	[9.27]	0.77	[4.85]					0.50
1,297	0.00	[−0.14]			0.84	[5.41]	0.00	[3.34]			0.46
1,297	0.00	[−0.21]			0.62	[3.77]			1.77	[5.31]	0.52
1,297	0.00	[−0.21]			0.61	[3.76]	0.00	[1.54]	1.76	[5.28]	0.52
1,297	−0.01	[−0.29]	0.002	[6.91]	0.59	[3.54]	0.00	[1.32]	1.55	[4.54]	0.55

Cross-sectional regressions of excess event returns on the contribution of each stock to the risk of the arbitrage portfolio and net purchases normalized by measures of liquidity

$$r_{it*} = a + b\Delta X_i + c(\Sigma\Delta X)_i + d(\Delta X_i / \overline{Vol_i}) + e(\Delta X_i / MV_i) + \eta_{it*}.$$

The dependent variable is the excess return during the event week. The independent variables are net purchases ΔX_i, expressed in yen, risk-adjusted net purchases $\Sigma\Delta X_i$, net purchases divided by average trading volume $(\Delta X_i / \overline{Vol_i})$, and net purchases divided by the market value of that stock $(\Delta X_i, / MV_i)$. Panel A performs the regression using data on all securities affected by the index redefinition. This covers 30 additions, 30 deletions, and 195 remainders. Panel B performs the regression using additions only. Panel C and Panel D analyze the remainders and deletions, respectively. Panel E studies all securities in the sample, which includes the 255 securities experiencing weight changes and the 1,042 securities not directly affected by the event. The table does not include a separate panel for unaffected securities because net purchases are zero for each of these stocks. Standard errors allow for cross-sectional correlation.

The coefficient falls to 0.95 when I control for liquidity, as proxied by market capitalization. Nevertheless, controlling for either measure of liquidity, arbitrage risk remains a significant determinant of event returns.

Panels B, C, and D repeat these regressions for the additions, deletions, and remainders separately. Each panel separately confirms the results in Panel A—the primary determinant of event returns is the contribution of the demand shock of that stock to the total risk of the arbitrage portfolio, and liquidity controls make virtually no difference.

The last panel estimates results for the combined sample of 255 affected and 1,042 unaffected securities. Regression coefficients d and e are identified off variation in the demand shocks of the affected stocks. These results again confirm that the contribution of the demand shock to arbitrage portfolio risk is the primary cross-sectional determinant of event returns.

4.5. Post-event returns

The model predicts that event returns should be reversed at a rate proportional to the initial event returns. This prediction holds for both the securities directly affected by the event and for the unaffected securities. The post-event reversal is a simple consequence of event returns reflecting expected future profits of arbitrageurs who initially absorbed the demand.

Fig. 6 provides some justification for the claim that, in the cross-section, post-event returns are negatively related to event returns. Panel A plots the buy-and-hold ten-week post event excess return for each affected stock against the return during the event week. This return is defined to be the raw ten-week buy-and-hold return minus beta times the buy-and-hold market return. Additions, deletions, and remainders are marked separately. The figure shows a negative linear relationship between post-event excess returns and event excess returns. Close inspection reveals that this pattern is borne out within the additions, deletions, and remainders separately.

Panel B plots the cumulative ten-week post-event excess return against the return during the event week for both affected and unaffected stocks. The figure shows that the negative linear relationship between post-event excess returns and event excess returns for stocks affected by the Nikkei inclusion also holds for the stocks not directly affected by the event, although there is more noise.

Table 7 tests the relationship between post-event and event returns shown in Fig. 6. For each week after the event, I regress the cumulative k-week post-event excess return on the excess event return for that stock

$$R_{it^*,it^*+k} = \lambda_k r_{it^*} + \eta_{ik}. \tag{26}$$

The coefficient λ_k is interpreted simply as the fraction of excess event returns that have reverted by week k. I show estimates for one, two, three, four, five, ten, 15, 20, and 40 weeks after the event, with the regression estimated on both the entire set of affected securities and on the additions, deletions, and remainders separately. The first panel shows that 32% of the initial excess event returns were reversed in the first
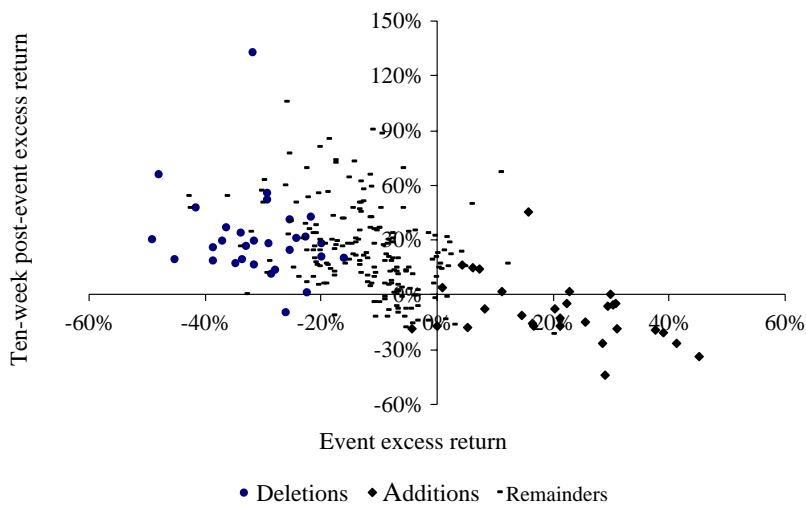
(A) Post-event returns, affected securities
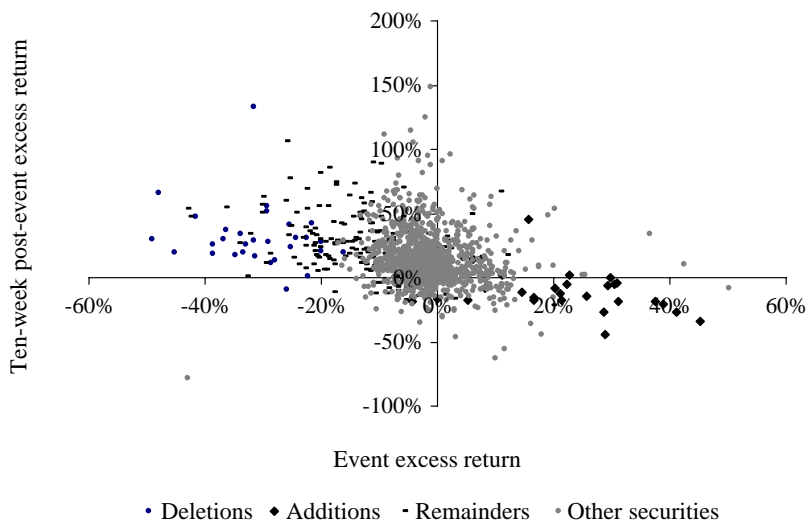
(B) Post-event returns, affected and unaffected securities

Fig. 6. Post-event excess returns. The figure plots buy-and-hold ten-week post-event excess returns for the 30 additions, 30 deletions, 195 remainders, and 1,042 unaffected securities against the excess return during the event week. Post-event excess returns are defined as the buy-and-hold return minus beta times the buy-and-hold return on the Tokyo Stock Exchange value-weighted index (TOPIX), where beta is computed using a market model. The horizontal axis is the excess event return. In Panel (A), the vertical axis is the ten-week post-event excess return and the sample includes only securities directly affected by the event. In Panel (B), the vertical axis is the ten-week post-event excess return and the sample is expanded to include 1,042 securities that did not experience weight changes during the event. Additions are marked with diamonds, deletions with filled circles, remainders with dashes, and all other securities with crosses.

Table 7
Post-event returns

Weeks after event	All affected securities (N = 255)				Additions (N = 30)				Deletions (N = 30)				Remainders (N = 195)				All unaffected securities (N = 1,042)				All securities (N = 1,297)			
(k)	a_k	[t]	λ_k	[t]	a_k	[t]	λ_k	[t]	a_k	[t]	λ_k	[t]	a_k	[t]	λ_k	[t]	a_k	[t]	λ_k	[t]	a_k	[t]	λ_k	[t]
1			−0.32	[−2.36]			−0.30	[−3.14]			−0.05	[−0.28]			−0.51	[−2.69]			0.04	[0.37]			−0.21	[−1.73]
2			−0.44	[−2.28]			−0.26	[−1.91]			−0.22	[−0.92]			−0.66	[−2.44]			−0.08	[−0.51]			−0.33	[−1.92]
3			−0.55	[−2.31]			−0.46	[−2.73]			−0.27	[−0.92]			−0.77	[−2.33]			−0.21	[−1.12]			−0.45	[−2.12]
4			−0.66	[−2.40]			−0.58	[−3.00]			−0.42	[−1.25]			−0.84	[−2.21]			−0.36	[−1.63]			−0.57	[−2.33]
5			−0.59	[−1.92]			−0.70	[−3.22]			−0.30	[−0.80]			−0.74	[−1.74]			−0.35	[−1.43]			−0.52	[−1.90]
10			−1.19	[−2.73]			−0.48	[−1.56]			−0.98	[−1.82]			−1.59	[−2.63]			−0.92	[−2.62]			−1.10	[−2.87]
15			−0.96	[−1.80]			−0.68	[−1.81]			−0.84	[−1.28]			−1.14	[−1.55]			−0.76	[−1.78]			−0.90	[−1.91]
20			−1.11	[−1.81]			−0.39	[−0.91]			−0.99	[−1.31]			−1.45	[−1.70]			−0.84	[−1.69]			−1.03	[−1.89]
40			−0.94	[−1.08]			−0.76	[−1.24]			−0.72	[−0.67]			−1.16	[−0.96]			−0.67	[−0.96]			−0.86	[−1.12]
1	0.01	[0.22]	−0.31	[−3.85]	−0.03	[−1.08]	−0.19	[−2.72]	−0.16	[−2.93]	−0.54	[−4.85]	0.00	[−0.14]	−0.53	[−5.58]	−0.00	[−0.22]	0.03	[0.37]	−0.00	[−0.13]	−0.22	[−2.58]
2	0.03	[0.75]	−0.36	[−3.17]	0.00	[−0.11]	−0.25	[−2.49]	−0.14	[−1.72]	−0.63	[−4.01]	0.02	[0.57]	−0.54	[−4.01]	0.03	[0.94]	0.00	[−0.04]	0.03	[0.93]	−0.25	[−2.11]
3	0.02	[0.45]	−0.49	[−3.53]	0.01	[0.25]	−0.51	[−4.19]	−0.09	[−0.90]	−0.53	[−2.76]	0.01	[0.26]	−0.71	[−4.26]	0.04	[1.06]	−0.11	[−0.83]	0.03	[0.96]	−0.35	[−2.37]
4	0.02	[0.35]	−0.60	[−3.79]	−0.02	[−0.32]	−0.51	[−3.68]	−0.02	[−0.18]	−0.48	[−2.17]	0.02	[0.27]	−0.77	[−4.02]	0.04	[1.04]	−0.24	[−1.58]	0.04	[0.92]	−0.46	[−2.71]
5	0.00	[0.08]	−0.58	[−3.25]	0.00	[0.06]	−0.71	[−4.57]	0.02	[0.17]	−0.24	[−0.96]	0.01	[0.12]	−0.70	[−3.29]	0.02	[0.41]	−0.30	[−1.74]	0.02	[0.35]	−0.47	[−2.49]
10	0.12	[1.30]	−0.85	[−3.38]	0.02	[0.26]	−0.56	[−2.56]	0.14	[0.78]	−0.57	[−1.61]	0.12	[1.35]	−0.98	[−3.24]	0.13	[2.08]	−0.55	[−2.24]	0.13	[1.98]	−0.74	[−2.75]
15	0.04	[0.39]	−0.84	[−2.72]	0.02	[0.15]	−0.74	[−2.75]	0.00	[−0.02]	−0.85	[−1.98]	0.04	[0.34]	−0.96	[−2.58]	0.07	[0.93]	−0.56	[−1.87]	0.07	[0.84]	−0.71	[−2.16]
20	0.11	[0.86]	−0.79	[−2.23]	0.06	[0.47]	−0.62	[−1.98]	0.14	[0.57]	−0.57	[−1.14]	0.10	[0.82]	−0.93	[−2.17]	0.12	[1.37]	−0.49	[−1.42]	0.12	[1.30]	−0.68	[−1.80]
40	0.05	[0.29]	−0.79	[−1.58]	0.05	[0.26]	−0.94	[−2.13]	0.03	[0.10]	−0.62	[−0.88]	0.07	[0.37]	−0.83	[−1.37]	0.00	[0.03]	−0.66	[−1.36]	0.01	[0.08]	−0.83	[−1.55]

Estimates from repeated cross-sectional regressions of post-event excess returns on the excess event return and a constant. Excess returns are defined as raw returns minus beta times the return on the Tokyo Stock Exchange value-weighted index (TOPIX) during the same period. Beta is estimated on three years of prior returns. Denoting the event week by t^*, event returns by r_{it^*}, and post-event excess returns between week t^* and $t^* + k$ as R_{it^*,it^*+k}, the table reports estimates from

$$R_{it^*,it^*+k} = \lambda_k r_{it^*} + \eta_{ik}$$

and

$$R_{it^*,it^*+k} = a_k + \lambda_k r_{it^*} + \eta_{ik}$$

estimated for $k = 1, 2, 3, 4, 5, 10, 15, 20,$ and 40 weeks after the event. In each regression, the independent variable is the excess event return. The bottom panel estimates also include a constant on the right-hand side. Standard errors allow for cross-sectional correlation.

week after the event. After five weeks, 59% of the initial excess event returns had been reversed; after ten weeks, 119% of the excess event returns were reversed. This pattern is confirmed among the additions (48% reversion after ten weeks), deletions (98% reversion after ten weeks), and remainders (159% reversion after ten weeks) separately. The pattern also appears among the group of unaffected securities. The table shows that 92% of their excess returns during the event week were reversed within ten weeks of the event (110% if reversion is estimated simultaneously for both affected and affected securities).

Eq. (26) does not include a constant term, imposing strict linearity between event and post-event excess returns. As a result, it does not allow for changes in security prices for each group of stocks related to news, such as market wide information driving all stock prices up or down. Adding a constant term eliminates this concern but also eliminates information about the average excess return for each group of securities, while preserving cross-sectional variation within each group.

The bottom panel of Table 7 shows estimates of the regression

$$R_{it^*+1,it^*+k} = a_k + \lambda_k r_{it^*} + \eta_{ik}. \tag{27}$$

The λ_k estimates now reflect reversion of cross-sectional variation within each group of securities and do not capture the average reversion of the group of additions, deletions, and remainders. Nevertheless, the results hold as before. For the group of all affected securities, 31% of cross-sectional variation in returns is reversed within one week of the event, and 85% within ten weeks of the event. This pattern again holds within the additions, deletions, and remainders, with 56%, 57%, and 98% of event returns reversed, respectively, within ten weeks of the event. This pattern also holds for the unaffected securities, with 55% of their excess event returns reversed within ten weeks of the event.

Fig. 7 traces the time series of estimates of λ_k from Eq. (26), estimated on the entire sample of affected securities. The figure shows that event returns reverted at a faster pace during the first four weeks after the event, after which the rate of reversion slowed. The reversion appears to be complete ten to 20 weeks after the event.

4.6. Post-event returns and arbitrage risk

The final set of tests verifies that the negative relationship between event and post-event excess returns comes from the fact that arbitrageurs are rewarded for risk taken during the event week. One might reasonably infer this to be true from the fact that event returns are positively correlated with contribution to arbitrage risk and that post-event returns are negatively correlated with event returns.

Table 8 tests the relationship between arbitrage risk taken by arbitrageurs during the event week and post-event excess returns at different horizons. I estimate Eq. (27) substituting arbitrage risk for event returns on the right-hand side of the regression

$$R_{it^*,it^*+k} = a_k + \lambda_k (\Sigma \Delta X)_i + \eta_{ik}. \tag{28}$$

The top four columns show these results for the sample of 255 stocks affected by the redefinition. Post-event excess returns are negatively related to risk-adjusted
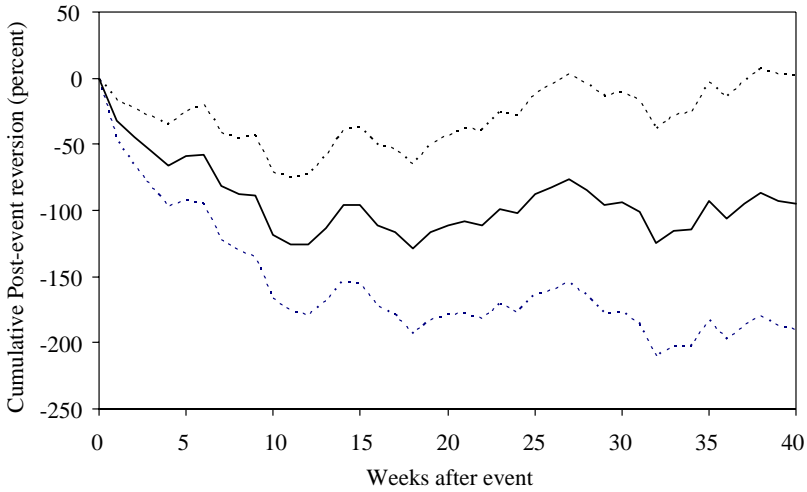
Fig. 7. Tracing the speed of post-event reversion. The figure plots estimates of the fraction of initial excess event returns that are reversed by week k. These estimates come from repeated cross-sectional regressions of post-event excess returns on the initial excess event return given in Table 7. The figure indicates confidence intervals given by ± 2 standard errors.

purchases, with the magnitude of the coefficient increasing with the post-event horizon. Reversion appears to stabilize within about ten weeks.

The remaining columns in Table 8 repeat these regressions for the additions, deletions, and remainders. These results confirm the same trend-post-event excess returns are negatively related to arbitrage risk, with the coefficient increasing in magnitude as the post-event horizon is extended. Although these results display varying degrees of statistical significance, it is remarkable that 13% of the cross-sectional variation in stock returns 20 weeks after the event can be explained by the contribution of these stocks to the risk of an arbitrage portfolio formed on April 21st.

The two bottom right-hand panels of results in Table 8 show that the negative relationship between post-event excess returns and the contribution to the risk of the arbitrage portfolio also holds for the 1,042 securities that did not experience weight changes during the event and for the combined sample of these stocks with the 255 stocks that experienced weight changes. Remarkably, yet consistent with the model, this means that the Nikkei redefinition continued to affect expected returns for most stocks in the market long after the event had passed.

5. The profitability of arbitrage strategies

While the reversion documented in this paper is consistent with positive expected returns to arbitrage, little has been said thus far about the link between post-event reversion and the ex post profitability of arbitrage strategies during the Nikkei 225 rebalancing.

Table 8
Post-event returns and arbitrage risk

Weeks after event	All affected securities ($N = 255$)					Additions ($N = 30$)					Deletions ($N = 30$)				
(k)	a_k	[t]	λ_k	[t]	R^2	a_k	[t]	λ_k	[t]	R^2	a_k	[t]	λ_k	[t]	R^2
1	0.02	[0.60]	−0.21	[−1.44]	0.05	−0.07	[−2.57]	0.02	[0.14]	0.00	0.04	[0.51]	0.15	[0.41]	0.00
2	0.04	[1.08]	−0.25	[−1.26]	0.08	−0.06	[−1.44]	0.06	[0.27]	0.01	0.06	[0.48]	−0.01	[−0.01]	0.00
3	0.02	[0.38]	−0.51	[−2.05]	0.20	−0.07	[−1.51]	−0.36	[−1.25]	0.20	0.23	[1.52]	0.57	[0.90]	0.02
4	0.01	[0.11]	−0.70	[−2.44]	0.27	−0.11	[−1.92]	−0.37	[−1.11]	0.16	0.27	[1.56]	0.53	[0.72]	0.01
5	−0.01	[−0.23]	−0.71	[−2.21]	0.23	−0.12	[−1.90]	−0.50	[−1.34]	0.17	0.41	[2.12]	1.20	[1.47]	0.08
10	0.09	[1.03]	−1.10	[−2.44]	0.21	−0.07	[−0.79]	−0.45	[−0.87]	0.11	0.45	[1.66]	0.53	[0.46]	0.00
15	0.01	[0.13]	−1.06	[−1.92]	0.18	−0.10	[−0.96]	−0.66	[−1.03]	0.20	−0.03	[−0.10]	−1.14	[−0.80]	0.01
20	0.09	[0.78]	−0.93	[−1.45]	0.11	−0.05	[−0.37]	−0.35	[−0.48]	0.06	0.25	[0.64]	−0.27	[−0.17]	0.00
40	0.01	[0.05]	−1.14	[−1.26]	0.13	−0.10	[−0.54]	−1.02	[−0.98]	0.24	−0.12	[−0.23]	−1.35	[−0.58]	0.01

Weeks after event	Remainders ($N = 30$)					Unaffected securities ($N = 30$)					All securities ($N = 1,297$)				
(k)	a_k	[t]	λ_k	[t]	R^2	a_k	[t]	λ_k	[t]	R^2	a_k	[t]	λ_k	[t]	R^2
1	0.04	[1.34]	−0.23	[−1.41]	0.04	0.00	[−0.02]	0.34	[1.79]	0.08	0.00	[0.10]	−0.06	[−0.38]	0.00
2	0.06	[1.75]	−0.21	[−0.93]	0.03	0.03	[1.14]	0.41	[1.53]	0.10	0.03	[1.18]	−0.04	[−0.20]	0.00
3	0.04	[0.89]	−0.52	[−1.83]	0.14	0.04	[1.11]	0.01	[0.04]	0.00	0.04	[1.04]	−0.28	[−1.03]	0.05
4	0.03	[0.63]	−0.67	[−2.05]	0.17	0.04	[1.02]	−0.33	[−0.87]	0.02	0.04	[0.91]	−0.48	[−1.56]	0.09
5	0.01	[0.20]	−0.72	[−1.99]	0.14	0.02	[0.34]	−0.55	[−1.30]	0.05	0.01	[0.26]	−0.58	[−1.69]	0.11
10	0.12	[1.39]	−1.10	[−2.13]	0.12	0.13	[2.00]	−1.02	[−1.69]	0.07	0.12	[1.90]	−0.97	[−1.99]	0.12
15	0.05	[0.49]	−0.91	[−1.45]	0.08	0.06	[0.83]	−1.23	[−1.66]	0.10	0.06	[0.74]	−0.99	[−1.66]	0.12
20	0.13	[1.11]	−0.76	[−1.05]	0.04	0.12	[1.36]	−0.63	[−0.74]	0.02	0.12	[1.30]	−0.75	[−1.09]	0.05
40	0.04	[0.26]	−1.08	[−1.05]	0.07	−0.01	[−0.06]	−1.45	[−1.20]	0.07	−0.01	[−0.04]	−1.29	[−1.32]	0.11

Estimates from repeated cross-sectional regressions of post-event excess returns on the contribution of each stock to the risk of the arbitrage portfolio and a constant. Excess returns are equal to raw returns minus beta times the return on the Tokyo Stock Exchange value-weighted index (TOPIX) during the same period. Beta is estimated on three years of returns data prior to the event. Denoting the event week by t^*, risk-adjusted net purchases by $(\Sigma \Delta X)_i$, and post-event excess returns between week t^* and $t^* + k$ as R_{it^*,it^*+k}, the table reports estimates from

$$R_{it^*,it^*+k} = a_k + \lambda_k (\Sigma \Delta X)_i + \eta_{ik}$$

estimated for $k = 1, 2, 3, 4, 5, 10, 15, 20,$ and 40 weeks after the event. Standard errors allow for cross-sectional correlation.

In single event studies, calculation of arbitrage profits requires an understanding of how arbitrageurs hedge their short positions in additions or long positions in deletions. In the Nikkei 225 redefinition, calculation of the arbitrage portfolio is trivial. Because net purchases of the additions were exactly offset by net sales of the deletions and remainders, arbitrageurs simply accommodated the entire demand vector.[12] I consider the profits of two similar strategies. The first strategy simply

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[12]This was confirmed by the author in an interview with Taro Hornmark, an arbitrageur who participated in the event.
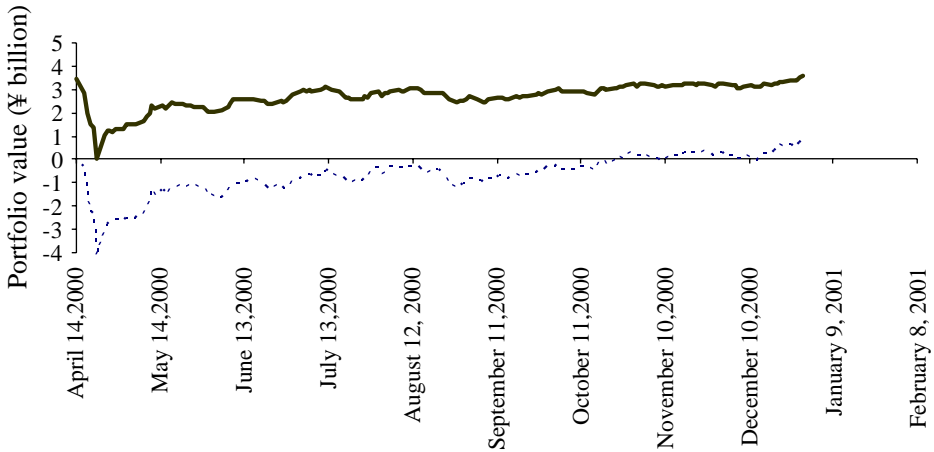
Fig. 8. Arbitrage profits. The figure shows the buy-and-hold value of the portfolio that is short 30 additions and long 30 deletions and 195 remainders in proportion to index weights based on closing prices on Friday, April 21, 2000. On this day, the portfolio is self-funded (has value zero) as the cost of the deletions and remainders exactly offset the proceeds from the additions. The portfolio corresponds to the holdings of an arbitrageur who absorbed 1% of the total demand induced by institutional rebalancing. The dotted line below shows the buy-and-hold value of the portfolio of an arbitrageur who absorbed 1% of total demand on April 14, the day the redefinition was announced. The demand shock in this case is calculated based on prices on that day. The vertical axis is ¥ billion (approximately ¥1 billion = US$9 million). Institutional funds linked to the Nikkei are assumed to equal ¥2.4 trillion.

accommodates the demand shock on the date of the announcement (April 14), based on expected rebalancing during that week. Fig. 8 shows the buy-and-hold value of this portfolio, indicated with a dotted line. It is short 30 additions and long 30 deletions and 195 remainders in proportion to index weights based on prices on Friday, April 14, 2000. On this day, the cost of the deletions and remainders exactly offsets the proceeds from the additions; thus the value of the portfolio is zero. The profits indicated on the vertical axis correspond to those of an arbitrageur who absorbed 1% of the total demand induced by institutional rebalancing, under the assumption that institutional funds linked to the Nikkei equal ¥2.4 trillion (Fukushima, 2000). The minimum value occurs on April 21, at which point the portfolio has declined in value by ¥4.17 billion.

The second strategy corresponds to the holdings of the arbitrageurs in the model, who take short positions against mispricing based on post-demand shock prices. This portfolio is short 30 additions and long 30 deletions and 195 remainders in proportion to index weights based on prices on Friday, April 21, 2000 and has zero value on this day. The figure shows the stunning profitability of this strategy—an arbitrageur who accommodated 1% of total demand based on these prices would have profits of ¥3 billion in less than 15 weeks.

The profitability of arbitrage strategies during the event occurred at the expense of institutions that were forced to trade at April 21 prices. The figure reveals that if these institutions had been willing to wait ten weeks or more after the event before

rebalancing, they would have avoided a loss of ¥3 billion. Because the figure represents only 1% of total estimated rebalancing, this implies a wealth transfer of approximately ¥300 billion to arbitrageurs. Ultimately, this raises a question about the rationality of tracking the Nikkei 225: Why would investors be willing to pay such a high price simply to match the index so precisely? While index fund managers have a clearly specified objective, their behavior must arise from strict contracts with investors. Although this paper is silent on the optimality of these contracts, they potentially limit agency costs between investors and fund managers and could provide additional benefits such as lower trading costs. If rebalancings of the magnitude experienced in April 2000 are rare, index contracts could be optimal ex ante. However, because the magnitude and the effects of the Nikkei rebalancing were unprecedented, one can reasonably speculate that investors would have done better to force index managers to rebalance several months later. Not unsurprisingly, following the redefinition, the popularity of the Nikkei 225 as a benchmark declined relative to the value-weighted TOPIX index (Maeda, 2000).

The magnitude of the wealth transfer to arbitrageurs raises another related question: If the rewards to arbitrage were so high, why were more investors in the market not willing to act as arbitrageurs? In the model, the quantity of arbitrageurs is fixed, a reasonable assumption for the short run. The fixed quantity ensures that when demand shocks are large, event and post-event returns are high. In the Nikkei event, entry was surely limited by the fact that most invested capital was effectively prohibited from shorting. Second, because the period in question occurred before a national holiday, some potential arbitrageurs could have been unavailable during this time. Thus the evidence in this paper is consistent with the Merton's (1987) claim that "on the time scale of trading opportunities, the capital stock of dealers, market makers and traders is essentially fixed." Modeling the dynamics of capital reallocation between various investing activities is worthy of further research but takes me away from the objectives in this paper.

6. Conclusions

This paper develops a framework to analyze demand curves for multiple risky securities at extended horizons when there are limits-to-arbitrage. A simple model describes the path of asset prices following an unexpected event in which many securities simultaneously experience potentially different changes in investor demand. The model offers several novel cross-sectional and time- series predictions. First, following a demand shock, I predict that the vector of event returns is proportional to the product of the covariance of fundamental risk of these securities and the vector of demand shocks. Put simply, security prices change in proportion to their contribution to the total risk of a diversified portfolio. Second, I predict price changes for securities that are not directly affected by demand shocks but are correlated with securities undergoing changes in demand. Specifically, the theory shows that the prices of these securities will be affected as arbitrageurs use them for hedging. Third, I predict a negative linear relationship between post-event returns

and the initial return associated with the change in demand. Thus, the initial event return is the change in price necessary such that arbitrageurs who accommodate the demand shock have positive expected returns following the event.

I test these predictions using data from a unique redefinition of the Nikkei 225 index in Japan, in which 255 stocks simultaneously experienced significant changes in index investor demand, causing more than ¥2,000 billion of trading in one week. I find a significant relation between event returns and the contribution of each demand shock to the risk of a diversified portfolio. Over 60% of the cross-sectional variation in event returns can be explained by this single variable. As predicted, I also find a positive relation between the returns of 1,042 securities not experiencing demand shocks and the change in their contribution to portfolio risk. Finally, I find that post-event returns are negatively related to the initial event return in the cross-section. In summary, all of the model's predictions are confirmed by the data.

Although the theoretical framework is particularly suited to the unusual event studied in this paper, it can be easily applied to any setting in which one or more securities experience a change in investor demand. Demand shocks arising from index arbitrage, swap sales, and portfolio restructurings share the feature that they involve the simultaneous purchase (or sale or both) of multiple stocks. A promising potential avenue for future research is to understand the cross-section of returns associated with demand from these types of trades. The multi security framework applied in this paper could also be useful for interpreting recent research linking changes in investor sentiment to the broad cross-section of U.S. stock returns (e.g., Baker and Wurgler, 2003).

In addition to verifying the cross-sectional predictions of the model, the April 2000 redefinition of the Nikkei 225 is strong evidence on the limits-to-arbitrage. The additions gained 19%, the deleted stocks fell by 32%, and the remaining 195 stocks fell by 13%. At least half of these returns were reversed during the subsequent 20 weeks. Thus an event that conveyed no economic news transferred more than ¥300 billion to arbitrageurs willing to wait ten to 20 weeks before rebalancing. Although arbitrageurs took considerable risk, the rewards were handsome by any measure.

Appendix A. Model with proofs

The capital market includes N risky securities in fixed supply with supply vector given by Q. The risk-free asset is in perfectly elastic supply with net return normalized to zero. The information flow regarding the liquidating dividend $D_{i,T}$, paid at time T, is given by

$$D_{i,t} = D_{i,0} + \sum_{s=1}^{t} \varepsilon_{i,s} \quad \text{for all } i \tag{29}$$

in which the information shocks $\varepsilon_{i,t}$ are independently and identically over time and are normal with zero mean and covariance matrix Σ.

Arbitrageurs maximize exponential utility of next period wealth subject to a wealth constraint

$$\max_N \mathrm{E}_t[-\exp(-\gamma W_{t+1})],$$

$$\text{such that } W_{t+1} = W_t + N_t'[P_{t+1} - P_t]. \tag{30}$$

W_t, P_t, and N_t are arbitrageurs' wealth, the vector of security prices, and the arbitrageur demand vector at period t, respectively.

First, solve for prices and returns when $t < t^*$. Demand is given by

$$N_t = \frac{1}{\gamma}[Var_t(P_{t+1})]^{-1}(\mathrm{E}_t(P_{t+1}) - P_t). \tag{31}$$

In the covariance stationary equilibrium, $Var_t(P_{t+1}) = V$. Then

$$P_t = \mathrm{E}_t(P_{t+1}) - \gamma V Q. \tag{32}$$

Iterating forward and substituting $\mathrm{E}_t(P_T) = \mathrm{E}_t(D_T)$, results in

$$P_t = \mathrm{E}_t(D_T) - (T - t)\gamma V Q. \tag{33}$$

Now,

$$P_{t+1} - \mathrm{E}_t(P_{t+1}) = \mathrm{E}_{t+1}(D_T) - \mathrm{E}_t(D_T)$$
$$= \varepsilon_{t+1}. \tag{34}$$

Multiplying both sides by their transposes, it follows that the expected future variance of prices, V, is equal to the covariance matrix of fundamentals, Σ. Substituting back into the equation for prices, this gives

$$P_t = \mathrm{E}_t(D_T) - (T - t)\gamma\Sigma Q. \tag{35}$$

Consider the effects of a permanent shock $u(N \times 1)$ to the supply of net assets. Substituting $(Q - u)$ for Q in the above,

$$P_{t^*} = \mathrm{E}_{t^*}(D_T) - (T - t^*)\gamma\Sigma(Q - u). \tag{36}$$

The event return is given by

$$P_{t^*} - P_{t^*-1} = \mathrm{E}_{t^*}(D_T) - \mathrm{E}_{t^*-1}(D_T) - (T - t^*)\gamma\Sigma(Q - u) + (T - t^* + 1)\gamma\Sigma Q$$
$$= \varepsilon_{t^*} + (T - t^*)\gamma\Sigma u + \gamma\Sigma Q. \tag{37}$$

The reversion of event returns between periods $(t^* + 1)$ and $(t^* + k)$ is given by

$$P_{t^*+k} - P_{t^*} = \mathrm{E}_{t^*+k}(D_T) - \mathrm{E}_{t^*}(D_T) - (T - t^* - k)\gamma\Sigma(Q - u)$$
$$+ (T - t^*)\gamma\Sigma(Q - u)$$
$$= \sum_{s=t^*+1}^{t^*+k} \varepsilon_s + k\gamma\Sigma(Q - u). \tag{38}$$

The expected reversion is thus

$$\mathrm{E}_{t*}\Delta P_{t*,t*+k} = k\gamma\Sigma(Q - u). \tag{39}$$

Finally, one can construct the conditional covariance of the vector of reversion returns with the vector of event returns, under the assumption that the $t-1$ conditional distribution of u is normal with zero mean.

$$\mathrm{cov}_{t*-1}(\Delta P_{t*}, \Delta P_{t*,t*+k}) = \mathrm{E}_{t*-1}\left([(T - t^*)\gamma\Sigma u] \cdot \left[\sum_{s=t*+1}^{t*+k} \varepsilon_s - k\gamma\Sigma u\right]'\right)$$
$$= -(T - t^*)k\gamma^2\Sigma \cdot \mathrm{E}(uu') \cdot \Sigma. \tag{40}$$

This matrix is negative semi definite. This implies that, for each stock i, its event returns are negatively correlated with its own post-event returns. Proposition 1 follows directly from Eqs. (37)–(40).

I now analyze the average cross-sectional covariance between the $N \times 1$ vector of event returns and the vector of reversion returns over any post-event window. Consider the ordinary least squares estimator of the regression of $\Delta P_{t*,t*+k}$ on ΔP_{t*}. The slope coefficient is given by

$$\beta_{\Delta P_{t*,t*+k},\Delta P_{t*}} = [Var_{t*-1}(\Delta P_{t*})]^{-1} Cov(\Delta P_{t*,t*+k}, \Delta P_{t*})$$
$$= \left[[(T - t^*)^2\gamma\Sigma u]'[(T - t^*)\gamma\Sigma u]\right]^{-1}\left[-k(T - t^*)\gamma^2\Sigma\mathrm{E}(u'u)\Sigma\right]$$
$$= \frac{-k}{(T - t^*)}. \tag{41}$$

Finally, I show that in the case in which the vector of demand shocks is proportional to the value-weighted portfolio, then event returns are proportional to beta. Recall that in the absence of a demand shock, returns in each period are given by

$$P_t - P_{t-1} = \varepsilon_t + \gamma\Sigma Q. \tag{42}$$

The market portfolio thus has returns in each period given by

$$MP_t - MP_{t-1} = [\varepsilon_t + \gamma\Sigma Q]'Q, \tag{43}$$

where MP_t denotes the time t value of the market portfolio. Define beta as the covariance of each security return with the market return. For each security i, beta is given by

$$\beta_i = \mathrm{cov}(\varepsilon_i, [\varepsilon_{t*} + \gamma\Sigma Q]'Q) = \mathrm{cov}(\varepsilon_i, \varepsilon'Q). \tag{44}$$

Thus the vector of betas is given by

$$\beta = \Sigma Q. \tag{45}$$

To analyze the effect of a demand shock that is proportional to the value-weighted market, substitute $u = \theta Q$ into the expression for event returns Eq. (37), where θ denotes any nonzero constant.

$$P_{t*} - P_{t*-1} = \varepsilon_{t*} + \gamma \Sigma Q[1 + (T - t^*)\theta] \tag{46}$$

or

$$P_{t*} - P_{t*-1} = \varepsilon_{t*} + \gamma[1 + (T - t^*)\theta]\beta. \tag{47}$$

Taking expectations yields the desired result.

Appendix B. Units of measurement

The model derives an expression for the change in price as a function of supply, expressed as a number of shares. However, the empirical results study returns as a function of the demand shock, expressed in yen. Here the mapping is derived from the units of the model to the units required for testing. Ignoring nonstochastic terms, recall that the vector of expected excess event returns is given by

$$E(P_{t*} - P_{t*-1}) = k\gamma \Sigma u, \tag{48}$$

where Σ is the covariance matrix of fundamental prices, u is the demand shock expressed as a number of shares, and P_{t*} and P_{t*-1} denote the price vectors, in yen. To get from here to returns, first write the above expression for a single security:

$$E(P_{it*} - P_{it*-1}) = k\gamma[\sigma_{[P_{it*}-P_{it*-1},P_{1t*}-P_{1t*-1}]}u_1 + \sigma_{[P_{it*}-P_{it*-1},P_{2t*}-P_{2t*-1}]}u_2 + \cdots$$
$$+ \sigma^2_{[P_{it*}-P_{it*-1}]}u_i + \cdots + \sigma_{[P_{it*}-P_{it*-1},P_{Nt*}-P_{Nt*-1}]}u_N]. \tag{49}$$

Divide by P_{it*-1} to get an expression for returns:

$$\frac{E(P_{it*} - P_{it*-1})}{P_{it*-1}} = \frac{k\gamma}{P_{it*-1}}[\sigma_{[P_{it*}-P_{it*-1},P_{1t*}-P_{1t*-1}]}u_1 + \sigma_{[P_{it*}-P_{it*-1},P_{2t*}-P_{2t*-1}]}u_2 + \cdots$$
$$+ \sigma^2_{[P_{it*}-P_{it*-1}]}u_i + \cdots + \sigma_{[P_{it*}-P_{it*-1},P_{Nt*}-P_{Nt*-1}]}u_N]. \tag{50}$$

The trick is to note that

$$P^2_{it*-1}\sigma^2_{\left[\frac{P_{it*}-P_{it*-1}}{P_{it*-1}}\right]} = \sigma^2_{[P_{it*}-P_{it*-1}]} \tag{51}$$

and

$$P_{it*-1}P_{jt*-1}\sigma_{\left[\frac{P_{it*}-P_{it*-1}}{P_{it*-1}},\frac{P_{jt*}-P_{jt*-1}}{P_{jt*-1}}\right]} = \sigma_{\left[P_{it*}-P_{it*-1},P_{jt*}-P_{jt*-1}\right]}. \tag{52}$$

Substituting Eqs. (51) and (52) into Eq. (50) and combining terms,

$$\frac{\mathrm{E}(P_{it+1} - P_{it})}{P_{it}} = k\gamma \begin{bmatrix} \sigma\left[\frac{P_{it+1}-P_{it}}{P_{it}}, \frac{P_{1t+1}-P_{1t}}{P_{1t}}\right] u_1 P_{1t} + \sigma\left[\frac{P_{it+1}-P_{it}}{P_{it}}, \frac{P_{2t+1}-P_{2t}}{P_{2t}}\right] u_2 P_{2t} + \cdots \\ +\sigma^2_{\left[\frac{P_{it+1}-P_{it}}{P_{it}}\right]} u_i P_{it} + \cdots \\ +\sigma\left[\frac{P_{it+1}-P_{it}}{P_{it}}, \frac{P_{Nt+1}-P_{Nt}}{P_{Nt}}\right] u_N P_{Nt} \end{bmatrix}.$$

(53)

Given that the units of u_i are the number of shares and P_{it*-1} is the price per share, $u_i P_{it*-1}$ are in yen. Thus, returns are linear in the quantity, expressed in yen, and the variance of returns. Applying this transformation to all of the relations in Proposition 1 and Proposition 2 yields the linear relations between returns, the vector of net purchases, and the covariance matrix of fundamental returns that are given in Proposition 3.

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