Optimal Tilts:
Combining Persistent Characteristic Portfolios

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Abstract

We examine the optimal weighting of four tilts in US equity markets from 1968 through 2014. We define a "tilt" as a characteristic-based portfolio strategy that requires relatively low annual turnover. This is a continuum, with small size, a very persistent characteristic, at one end of the spectrum and high frequency reversal at the other. Unlike low-turnover tilts, a full history of transaction costs is essential for determining the expected return of, and hence the optimal allocation to, less persistent, more turnover-intensive characteristics. The mean-variance optimal tilts toward value, size, and profitability are roughly equal to each other and equal to the optimal low beta tilt. Notably, the low beta tilt is not subsumed by the other three.

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I. Introduction

Systematic equity investing goes by many different names: rules-based investing, sorts, style, characteristic-based portfolios, factor investing, smart beta, alternative beta, and even *genius* beta.[1] Investors use characteristic-based portfolios in two ways. The first is to evaluate risk. Across multiple equity managers, an investor may monitor and manage intentional or unintentional exposures to one or more characteristics. The second is to generate return, by combining characteristics in a single portfolio, or by assembling multiple, single-characteristic portfolios.

We can draw another distinction among these investment strategies. We use the persistence of the characteristics themselves to split seven stock characteristics into two groups. Some are persistent "tilts." For example, small capitalization investing requires little annual trading, because small stocks this year are likely to have been small stocks last year, reflecting an annual autocorrelation of 0.97. Other strategies are higher frequency "trades." While we use two labels for simplicity, persistence is a continuum. Growth, momentum, and high frequency reversal require successively more frequent rebalancing, with annual autocorrelations of 0.30, 0.05, and 0.03. All of the extreme characteristics tend to appear among illiquid stocks, and thus high turnover requires a careful eye on implementation costs.

When is this categorization of systematic strategies important? It is not crucial for the evaluation of risk. Both tilts and trades can be used to assess contributions to portfolio risk. But the distinction is essential in portfolio construction. Forming mean-variance efficient portfolios, or assessing the incremental value of adding one characteristic portfolio to an existing set, requires that the portfolios under consideration be equally implementable. For example, suppose

[1] The consulting firm Segal Rogerscasey asks, "Genius Beta: Why Settle for Just Smart Beta," in its *Investment Focus* of June 2015.

the risk and gross return properties of a low beta portfolio could be roughly matched with a blend of a momentum portfolio and a high frequency reversal portfolio. Because the returns net of implementation costs, and therefore the capacity, of the low beta portfolio are much greater, the comparison of gross returns is unhelpful. While the gross returns of our tilts—all with an annual autocorrelation greater than 0.7—can be reasonably compared on an apples-to-apples basis, the gross returns of trades cannot. A more ambitious model, complete with a long history of transaction cost estimates, assets under management, and cash flows, is needed to arrive at a mean-variance optimal combination of single-characteristic portfolios. In this paper we focus narrowly on the optimal combination of tilts, which are likely the most relevant for large-scale equity investors, such as pension funds, endowments, and sovereign wealth funds.

We study what would have been the static, optimal tilts over the period 1968 through 2014 for an investor considering deviations from a benchmark of cash and a passive market portfolio of US stocks. We examine the risk and return properties of versions of the tilt portfolios that have been standardized to have zero market exposure and a volatility of 1% per month. The ideal balance of risk and return would have been achieved by dividing active tilts roughly equally: A 20% share to value, 26% to small size, 23% to high profits, and 24% to a low beta tilt. The remaining 7% is allocated to bond market factors. Notably, in an apples-to-apples comparison, the low beta tilt is not subsumed by other tilts. Rather, it is the second highest of the four.

The final allocations to the simple tilt portfolios, the market, and cash depend on the desired level of active risk, or tracking error. For example, $100 invested with a 2% active risk to a 60/40 equity/cash benchmark would have been optimally divided into cash of $31, an investment in a passive US stock portfolio of $69, and long-short (zero net investment) tilts to value ($10 long and short), size ($11), profit ($1.50), low beta ($10), and duration and credit ($3 and $0.30, respectively). Because the optimal tilts towards value, high profits, and especially low beta involve a reduction in exposure to the passive market portfolio, the optimal portfolio involves an increased allocation to equities from the benchmark 60/40 portfolio to 69/31.

II. The Implementation of Characteristic Portfolios

We start by choosing a standard set of characteristic portfolios, including one that is long lower beta stocks and short higher beta stocks. In this section, we describe the selection process, the measurement of beta, and—crucially—the relative ease of implementation. The output is four portfolios that we describe as tilts. These are implemented at modest rates of turnover, which we measure using the annual autocorrelation of characteristic values. While this is a continuum, we choose an arbitrary cutoff autocorrelation of 0.7 to separate the most persistent four from three other portfolios that have lower capacity because of their inherent high turnover. Finally, to this set, we add two easily implementable tilts from the fixed income market: one that captures duration, and the other that captures credit risk. The fixed income tilts are included primarily as controls, for their potential to capture risk in the cross-section of stock returns.

A. Choosing Characteristics

There is a wide array of firm characteristics to use in the prediction of stock returns. Here, we define an anomaly conventionally, as a deviation from the return predicted by the Capital Asset Pricing Model (CAPM). The CAPM is of course an imperfect theoretical model of stock returns, so these deviations can be interpreted either as missing risk factors or mispricings. These fall into several categories: Small, safe, value, conservative growth, and profitability.

Technical indicators, momentum and reversal—which rely only on past returns—round out a preliminary list.

Safe stocks, defined as low beta, have relatively high returns in Black, Jensen, and Scholes (1972) and more recently in Baker, Bradley, and Wurgler (2011) and Frazzini and Pedersen (2014). Similar results obtain with low volatility instead of beta in Ang, Hodrick, Xing, and Zhang (2006, 2009) and Blitz and Van Vliet (2007). Small stocks, defined as relatively low market capitalization, have higher-than-CAPM predicted returns in Banz (1981). Value stocks, defined as those with relatively low price/book have abnormally high returns in Rosenberg, Reid, and Lanstein (1985), Chan, Hamao, and Lakonishok (1991), and Fama and French (1992).

Profitable firms have higher average stock returns in Basu (1983), Haugen and Baker (1996), Cohen, Gompers, and Vuolteenaho (2002), Fama and French (2006), and Novy-Marx (2013). Returns after stock sales, IPOs, and SEOs are abnormally low, while returns after stock repurchases are abnormally high in Ritter (1991), Loughran and Ritter (1995), Ikenberry, Lakonishok, and Vermaelen (1995), Pontiff and Woodgate (2006), and Daniel and Titman (2006). Relatedly, firms with high accruals (Sloan, 1996), relatively high capital expenditures (Fairfield, Whisenant, and Yohn, 2003; Titman, Wei, and Xie, 2004; Xing, 2008), large growth in net operating assets (Hirshleifer, Hou, Teoh, and Zhang, 2004) or total assets (Cooper, Gulen, and Schill, 2008) also have abnormally low returns. We refer to these patterns collectively as conservative or low growth.

Stock returns exhibit momentum, in that firms with relatively high trailing returns have abnormally high average returns, and reversal over shorter horizons of a month or less in Jegadeesh (1990) and Jegadeesh and Titman (1993, 1995a, 1995b).

In U.S. data, all of these return predictors can be measured back to the early 1960s, and in many cases all the way back to the 1920s. The list that includes predictors with shorter histories is even longer, and is built on data on mutual fund and institutional holdings, governance, short selling, options, analyst recommendations and estimates, earnings announcement surprise, and more.

The goal of this paper is not to survey the vast array of potentially useful signals, but to analyze a simple and transparent subset that subsumes the themes in the long-history data. For that reason, we narrow our attention to an initial subset that includes the five factors in Fama and French (2015), as well as a simple implementation of momentum and one-month reversal from Jegadeesh and Titman (1993). Put another way, these are the six characteristics that Ken French labels as “factors” in his data library. The only factor from the data library that we leave out is the long-term reversal factor, which has received much less attention in the academic and practitioner literatures. We use each of these factors exactly in their canonical form. To this list, we add risk, measured with a trailing estimate of beta. The initial set of tilts and trades is listed in Table I, where the simple factor definitions are shown. It would be straightforward to extend the analysis to a larger list of factors, but this would require reducing the length of the time series for factors with limited history, and likely an additional aggregation exercise designed to narrow the larger set of characteristics to a smaller number of principal components, along the lines of Stambaugh, Yu, and Yuan (2012).

B. Measuring Beta

We choose among three different measures of beta by selecting the best predictor of realized risk. Notably, we do not aim to make the measure of beta more persistent or more implementable, just as we have made no such attempt with the other six candidates described above. The first measure of beta uses the traditional five years of monthly returns, following Baker, Bradley, and Wurgler (2011) and the definition of beta in Ken French's data library; the second uses five years of three-day overlapping returns; and the third uses the correlation estimate from the second method plus a one-year daily volatility, as in Frazzini and Pedersen (2014). To round out the list, we also examine the one-year daily volatility on its own. All are effective at spreading risk, as shown in Table II. We use the Fama and French method described in the next section to define the "high" and "low" beta portfolios.

The differences in realized beta between high and low portfolios, measured monthly, are 0.68, 0.77, and 0.78, respectively, and in all cases highly statistically significant. It is not hard to form portfolios of stocks with levels of standard deviation that are reliably below the overall market. (Daily volatility on its own is not quite as good as the best estimates of beta, but still a worthy contender, with a spread of 0.74.) We use the third approach, though the second and third produce nearly identical results. The key is using three-day returns to estimate correlations. This has the effect of lowering the average betas of small stocks, which are individually less likely to trade in synch with the market overall, because of lower levels of liquidity. As a result, the improved measures of beta are lower for smaller capitalization stocks. As a practical matter, this makes the portfolios in Table II somewhat harder to implement; but as we show in Section II.D below, it puts the beta tilt on par with the other characteristic tilts, like value and high profits, which have as much or more dispersion in smaller capitalization stocks as the three-day overlapping returns estimate of beta.

C. Forming Characteristic Portfolios

We settle on seven characteristics in Table I: Low Beta, Value (Fama-French HML), Small Size (SMB), High Profits (RMW), Low Growth (CMA), Momentum (MOM), and Reversal (STREV). We compute portfolio returns for each following the approach pioneered by Fama and French (1993), forming factor portfolios with some consideration implicitly given to implementation costs. We discuss the effects of implementation in the next section. For example, the Fama-French value factor divides the universe into six portfolios according to NYSE breakpoints: small and big value, small and big neutral, and small and big growth. At the end of each June, six portfolios are rebalanced with market capitalization weights within each one. The value factor portfolio is long equal amounts of the two value portfolios and short equal amounts of the two growth portfolios. Using NYSE breakpoints and value-weighting portfolios give the factor portfolio greater realism by giving less weight to tiny stocks.

Table III shows the performance of the seven factor portfolios over the period from 1968 through 2014. (The size factor is designed by Fama and French to be neutral to value.) All come directly from Ken French's data library except for the beta portfolio, which follows the Fama-French conventions with end-of-June rebalances and uses the estimate of beta shown in the third row of Table II. The first three columns show the average annualized monthly return, the annualized standard deviation, and the Sharpe ratio, which is the ratio of the average to the standard deviation. These annualized returns range from 1.8% for Low Beta to 7.9% for Momentum.

The next four columns show the market-neutral performance of the seven factor portfolios. These are the results of a regression of each factor portfolio on the excess return on the value-weighted market portfolio (Fama-French MKT). The average market-neutral monthly return, or alpha, is equal to the annualized intercept. The standard deviation is equal to the annualized standard deviation of the regression residuals. And, the Sharpe ratio is again the average divided by the standard deviation. For example, the low beta factor portfolio by construction has a very low beta, and so on a market-neutral basis, its performance is much stronger, with a market-neutral annualized return of 6.4% and a market-neutral Sharpe ratio of 0.62 versus raw values of 1.8% and 0.11. The average market return over Treasury bills was 5.6% over this period, so low betas enhance market-neutral performance. The performance of value, high profits, low growth, and momentum also improve, with negative market exposure taken into account, but to a much smaller extent. Meanwhile, the size and reversal factor portfolios have somewhat weaker performance on a market-neutral basis because on average they have positive market exposure. The market-neutral annualized returns range from 1.3% for Small Size to 8.6% for Momentum.

One important note: it is critical to form all of the characteristic portfolios the *same way*. For example, it is unreasonable to judge the returns on a long-only, large capitalization, low beta portfolio against the Fama-French-style long-short implementation of profits (CMA), with equal weights on small and large capitalization stocks. This is why we form the low beta characteristic portfolio using precisely the Fama-French methodology. It is long-short, and it blends beta tilts among both small and large stocks.

Mixing and matching can produce illogical conclusions. For example, using the identical measure of value but focusing on small stocks produces a portfolio that has a statistically positive alpha in Fama-French time series regressions. Using the identical measure of value but focusing on large stocks produces a portfolio that has a statistically negative alpha in Fama-French time series regressions. Similarly, long-short implementations in small stocks produce higher alphas than long-only implementations. All of these conclusions are silly. Controlling for value, value portfolios should not show positive or negative alphas, but because mispricings are generally stronger in small stocks, differences in portfolio construction, turnover, and liquidity can lead to more insidious conclusions that are as incorrect but harder to spot.

D. Implementation: Tilts versus Trades

Six of the seven market-neutral factor portfolios have Sharpe ratios that are higher than the market over this period. Size is the one exception. However, even though Fama and French design their factor portfolios to represent plausible trading strategies, the last three columns of Table III show that these strategies will differ considerably when it comes to real-world implementation. We perform three correlations, using Compustat data and the definitions from Ken French's data library. The first is the average annual autocorrelation of the underlying characteristics used to form the portfolio. These range from 0.97 for size (market capitalization) down to 0.03 for reversal (trailing one-month return), which map intuitively to portfolio turnover. An annually rebalanced size tilt requires close to zero turnover to maintain. Meanwhile, an annually rebalanced reversal portfolio requires a much higher rate of turnover. In the case of the monthly reversal and momentum factor portfolios, the turnover is greater than 100% per year.

A second challenge to implementing these trading strategies is liquidity. The third-to-last and second-to-last columns of Table III show the average cross-sectional rank correlation between each underlying characteristic and market capitalization, separately reported for stocks with above-median (+) and below median (–) characteristics. A negative number means that a positive tilt requires buying smaller than average stocks in the Fama-French implementation of these factor portfolios. With the natural exception of size, characteristic correlations with market cap are negative for both above-median and below-median companies, meaning that the largest stocks fall in the middle of the characteristic distribution and are neither bought nor sold short. (For below median characteristics, we negate the characteristic value, to show the correlation with market capitalization of taking the opposite side from the basic characteristic, e.g., high beta, growth, large size, low profits, and so on.) In combination with a low autocorrelation, this suggests that the implementation costs of momentum and reversal are high, and the Sharpe ratios in the third and seventh columns of Table III need to be adjusted materially.

We divide the group according to autocorrelation with the visual breakpoint that is apparent between the autocorrelation of profitability, at 0.72, and low growth, at 0.30. The liquidity demands in the tails of the characteristic portfolios are not noticeably different, with the exception of size which has an intuitive asymmetry. We take this to mean that the returns on the first four characteristic portfolios were, to a great extent, achievable, or similarly achievable, over the period from 1968 through 2014. The annual turnover is low enough not to materially change the gross returns in the first and fifth columns of Table III. We label these "tilts." Meanwhile, the last three characteristic portfolios require significant turnover, involving small, less liquid stocks, and an implementation shortfall with any material level of assets under management. This is not to say that these are unappealing strategies, but in an analysis of whether one tilt is subsumed by another, or an analysis of optimal allocations to these characteristic portfolios, implementation cannot safely be ignored. A careful analysis must include an assessment of the full time series of transaction costs, which will necessarily be dependent on the dollars to be invested, meaning that one size cannot fit all.

To be sure, transaction costs have fallen, so implementation costs are now more modest, but in the first half of the sample they were not trivial. And, as one might expect in a competitive market, this fall in transaction costs is accompanied by lower Sharpe ratios in the higher turnover strategies. The raw Sharpe ratio of the low growth (CMA) portfolio dropped from 0.76 in the first half of the sample to 0.54 in the second half. Meanwhile, momentum (MOM) dropped from 0.73 to 0.39, and reversal (STREV) from 0.94 to 0.23. We label these last three “trades.” Adjusting these for realistic trading costs is a useful exercise but beyond the scope of this paper.

The upshot is that we can use the returns in Table III to assess the overlapping risks of the seven characteristic portfolios but not the average returns. For example, suppose that the low beta portfolio could be partially mimicked with a combination of low growth and momentum stocks. From a risk perspective, this is interesting. The low beta portfolio could then be judged as somewhat likely to underperform at the same time as the high profits portfolio and the momentum portfolio. However, the relative attractiveness of these two alternatives—the low beta tilt versus a blend of momentum and low growth trades—cannot be evaluated using the gross-of-transaction cost returns. For this reason, we focus our evaluation of *risk* on Tilts and Trades, and our evaluation of *returns* on Tilts alone.

Interestingly, Li, Sullivan, and Garcia-Feijóo (2014) argue that the raw performance of low risk is less impressive after transaction costs are considered, but it is worth noting that their analysis focuses on more transient measures of risk. Table III shows, by contrast, that beta is perhaps the most implementable tilt of the group, with an autocorrelation of 0.88, and a combined size correlation in the tails of 0.31. It is materially more persistent than value and profitability, with the same liquidity demands. It is less persistent than size but mechanically requires a much more modest cap correlation on the long side. Consistent with this conclusion, Baker, Bradley, and Wurgler (2011) and Auer and Schuhmacher (2015) find strong results in the value-weighted top 1,000 stocks and the even the Dow 30, respectively. Growth, momentum, and reversal require much higher annual rebalancing.

E. Correlations with Beta

The final column in Table III shows how each of the seven tilts and trades correlates with beta in the cross-section. Low beta stocks, on average, have higher value scores. This lines up with the portfolio beta in the fourth column. The cross-sectional rank correlation with profits, growth, and reversal are essentially zero, despite these sorts producing a modest beta tilt in the fourth column. This suggests that generating a low beta tilt using trailing estimates of beta involves buying an entirely different set of stocks than tilts toward profits or trades that capitalize on low growth, even though the portfolio tilted towards higher profits or lower growth has a statistically significant covariance or portfolio beta. And, the cross-sectional correlation between the trailing estimate of beta with market capitalization and momentum go in the opposite direction of the portfolio betas, again indicating no practical overlap in the stock selection strategies.

F. Fixed Income

We also include two credit market portfolios, largely as controls. The first is the return on long-term government bonds to capture the effect of interest rate movements and the premium for bearing that risk, which we label Duration. The second is the difference between the return on investment grade corporate bonds minus the return on long-term government bonds to capture the effect of credit risk movements and the associated risk premium, which we label Credit.

Notably, the duration portfolio has been linked to the returns of low beta and profitable stocks in Baker and Wurgler (2012). We also remove the average market exposures in these portfolios, so the analyses in this paper can be considered as tilts away from a benchmark cash and equity portfolio, through characteristic and fixed income portfolios.

III. The Risks of Low Beta: Is Low Beta Subsumed by Value, Size, Profitability, and the Bond Market?

Before turning to the main exercise of computing mean-variance optimal tilts in the next section, we examine the incremental value of a low risk tilt. Empirical studies of risk and return date back at least to the 1970s and Black (1972), Jensen, Black, and Scholes (1972), and Haugen and Heins (1975). More recent work, including Fama and French (1992), Ang, Hodrick, Xing, and Zhang (2006, 2009), Blitz and van Vliet (2007), Baker, Bradley, and Wurgler (2011), Baker, Bradley, and Taliaferro (2014), and Frazzini and Pedersen (2014) use more updated data, global markets, other asset classes beyond equities, and a broader set of risk measures, including idiosyncratic risk. The upshot of all of these papers is that risk and return are at most weakly related.

Some scholars have challenged the practical relevance of these findings. It is not that the seminal papers got the empirics wrong, but rather that the results are subsumed by even more fundamental drivers of return, notably value and profitability. For example, Shah (2011) and Crill (2014) argue in white papers that the performance of risk-tilted portfolios comes from the correlation between value and beta, and much of that performance comes from the periods where the two characteristics align. Novy-Marx (2014) emphasizes profitability instead of value. A third paper by Bali, Brown, Murray, and Tang (2017) makes a more surprising claim that the low risk tilt is subsumed by the maximum daily return from the previous month. In contrast, we find that much of the risk and half of the return of low beta remains unexplained, in an analysis that puts low beta on an apples-to-apples "tilts" basis with value and profitability, and leaves aside higher turnover "trades" like the maximum daily return from the previous month.

A. The Risks of Low Risk

A first question is how much of the month-to-month variation in low beta returns is explained by other characteristic portfolios. The variance of the low beta returns is 107%^{2}, which is equal to the standard deviation of 10% in Table III squared. We decompose this variance into components explained by other characteristic portfolios in linear regressions in Table IV.

We start with univariate effects, regressing the low beta portfolio returns on the other characteristic portfolios. The coefficients show that, when low beta stocks underperform on a market-adjusted basis, so too do larger stocks, value stocks, profitable stocks, and stocks with lower asset growth. These are what one might intuitively call the more boring, and less risky stocks on average. In the fourth column of Table III, the betas of these portfolios line up correspondingly. While the market effects have been removed, the results in Table IV suggest that residual returns remain a common component across these various portfolios. Momentum and reversal explain less risk on average. Momentum is an interesting case. Unlike the more persistent characteristics, momentum tends to occasionally line up with high beta stocks—for example, in the late 1990s in a rapidly rising equity market—and with low beta stocks during a market correction like the fall of 2008 and the spring of 2009. So it is hard to think of momentum as a stable risk factor. Momentum instead inherits the risk of whatever characteristics have been in favor of late. For each factor, we compute a univariate R-squared. None of these factor portfolios on its own explains more than 17% of the risk of the low beta portfolio.

The fixed income effects are also intuitive. Low beta stocks are more ‘bond-like’ and investors may view them, rationally or not, as closer substitutes for long-term government bonds. This is a comparatively large effect, explaining 10% of the risk of the low beta portfolio. The effect of credit is smaller, but in the expected direction. Narrowing credit spreads might indicate a rise in risk appetite and hence weaker performance of low risk stocks.

The covariances of the six characteristic portfolios and two fixed income portfolios overlap. For example, high profit firms tend to grow more slowly and trade at lower multiples. So the sum of the univariate effects is more than the combined explanatory power. Column 9 of Table IV shows a multivariate attribution of the returns of the low beta portfolio. Most of the univariate effects carry over. Value stocks, large stocks, profitable stocks, slow growing stocks, and duration still explain the low beta returns as before. Momentum becomes slightly stronger statistically, while reversal remains weak. Credit changes sign, suggesting that once the returns of the other characteristic portfolios are taken into account, low beta stocks tend to perform better when credit spreads are widening, but this is very small by comparison. The multivariate regression allows us to put a point estimate on the ability of these seven portfolios to mimic the risks of low beta portfolios at 41%.

Even this 41% is not especially robust. It would be hard to form a reliable replicating portfolio. For example, momentum suffers a historic drawdown in 2009 that lines up with low beta, but otherwise there is no apparent correlation. A single episode of correlated poor or strong performance can overshadow what is otherwise a weak relationship.

B. The Returns of Low Beta

A second question is, how much of the average return of the low beta portfolio is explained by other characteristic portfolios? The average alpha of the low beta portfolio is 6.4%. Here, the takeaway is that 47% of this risk-adjusted return is explained by other persistent characteristic tilts. The remaining 53% is unexplained. We again perform this analysis with a set of univariate and multivariate regressions of the time series returns summarized in Table III.

The results are shown in Table V. This table takes the coefficients from Table IV and multiplies them by the market-neutral return of each strategy to measure the portion of the market-neutral low beta portfolio return that is explained by each characteristic tilt. For example, because value has an annualized market-neutral return of 5.7% per year and the low beta portfolio has a loading of 0.31 on this portfolio, the part of the low beta return that overlaps with value is 1.8%. The total annualized market-neutral return on the low beta tilted portfolio is 6.4%, so 1.8% represents 28% of the total alpha. Similar calculations can be done for high profits and duration, which overlap by 1.8% and 0.9%, respectively. The impact of size and credit are smaller. The multivariate regression takes into account the union of these overlapping portfolios. In all, the three characteristic tilts and the two fixed income portfolios have market-neutral returns that overlap 3.0% out of the 6.4% alpha for the low beta tilt, or 47% of the average alpha of the low beta portfolio.

It is important to note that we excluded the high turnover, high liquidity demand characteristic portfolios because their Sharpe ratios are not fully implementable, and so the mimicking portfolio approach that is implicit in these regressions would overstate the extent to which low growth, momentum, and reversal are able to explain the returns to low beta. The analysis of risk says that there is a considerable portion of low beta that cannot be captured by other means, and the analysis of return says that there is no sense in which the returns of low risk can be reproduced with similar risk and return characteristics with a portfolio of other tilts. At least 53% of the low risk anomaly remains after other tilts are taken into consideration. Going a bit further, the correlations at the level of characteristics provide additional emphasis. For example, the characteristic correlation of low beta with high profits is exactly zero, meaning that the overlapping risk and returns are not coming from holding the same stocks, but rather holding different stocks that have overlapping return patterns. This is an important note for capacity. Even if the return series were identical, splitting the tilt between high profits and low beta would economize on transaction costs, assuming that the price impact of trade is convex.

IV. Optimal Tilts, 1968-2014

We now turn our attention to computing the optimal tilts over the period from 1968 through 2014, with an exercise of simple mean variance analysis. The starting point is four equity tilts plus two fixed income portfolios. We use the monthly in-sample correlation matrix and portfolio covariances to measure risk. And we use the in-sample average returns to measure return. The question is what combination of these six portfolios would have produced the highest Sharpe ratio over this period.

A. Mean-variance analysis

Table VI shows the inputs to the analysis. Starting in the second column, we reproduce the average, market-neutral returns from Table III, and we also express these per unit of standard deviation in monthly returns. These range from –0.3% per standard deviation, per year for the credit portfolio to 2.2% for low beta. The last six columns show the in-sample correlations. The correlations will be familiar from the results of Table V, where we regressed the returns to the low beta portfolio on each of the other factor portfolios: The low beta tilts have a positive correlation in returns with value, profits, and duration, and a negative correlation with size and credit.

The optimal blend of these six portfolios produced an average return per monthly return standard deviation of 3.3%, indicating an annual Sharpe ratio of 1.0. The interesting part is the shares in the first column. A high allocation of the risk budget goes to the low beta tilt at 24%. Despite its low Sharpe ratio, the highest allocation is to the small size portfolio. The reasons for this are evident in the correlation matrix. Size is negatively correlated with all but credit, and it has a meaningfully large and negative correlation with both low beta and high profits. This means that the allocation to size, despite its own low Sharpe ratio, allows a greater tilt toward low beta and high profits in particular. Next is high profits at 23% and value at 20%, with much lower weights allocated to the two fixed income portfolios at 7% for duration and 0% for credit.

Figure 1 summarizes these allocations, and compares them to the shares when low growth is also included. We are inclined to categorize Low Growth as trade, given its high annual turnover of roughly 70% and its emphasis on small stocks in the extreme portfolios. As a result, its high Sharpe ratio, when returns are measured gross of implementation costs, may not be relevant for most investors with large assets under management. Nonetheless, growth has lower execution costs than momentum or reversal. Including Low Growth has the largest impact on Value. Like Fama and French (2015b), we find that the share allocated to Value goes to zero. The effect on beta is more modest, cutting the share from 24% to 11%. We interpret this range as a plausible confidence interval for beta, which bounds its role in an optimal portfolio of tilts. At low levels of assets under management and execution costs, 11% is appropriate. At high levels, higher shares are appropriate, given the very low cross-sectional correlation of beta with growth in Table II and the higher implementation costs that reduce the gross returns to growth relative to those of beta.

B. Some example implications

It is not worth attaching too much significance to the specific weights in Table VI. For example, with an equal allocation to the four tilts, the return falls by only one basis point. Another way of transforming these tilts is to consider how many dollars would have been devoted to each portfolio given an annual standard deviation or active risk versus a benchmark allocation to cash and equities.

We convert the shares in Table VI to a practical portfolio policy in Table VII. The final allocations to the simple tilt portfolios, the market, and cash depend on the desired active risk. For example, $100 invested with a 2% active risk to a 60/40 equity/cash benchmark would have been optimally divided into cash of $31, an investment in a passive US stock portfolio of $69, and long-short (zero net investment) tilts to value ($10 long and short), size ($11), profit ($13.50), low beta ($10), and duration and credit ($3 and $0.30, respectively). Because the optimal tilts towards value, high profits, and especially low beta involve a reduction in exposure to the passive market portfolio that is greater than the increase that comes from the tilt toward small size, the optimal portfolio involves an increased allocation to equities from the benchmark 60/40 portfolio.

As the active risk increases, the dollars allocated to the tilts rise from a $2.60 low beta tilt to produce a 0.5% standard deviation to a $51 low beta tilt to produce 10% standard deviation from the benchmark. At the same time, the allocation to equities rises from $60, to a leveraged $105 to offset the lower market exposure that comes from a low beta tilt. At low levels of active risk, leverage and short selling are not necessary. The optimal portfolio involves a slight underweight to higher beta stocks, a slight overweight to lower beta stocks, and a substitution of equities for cash. Meanwhile, at high levels of active risk, underweights turn to short positions, and cash turns to borrowing. These allocations could be implemented, at some cost in risk and return, with no short selling or leverage. Like the analysis of implementation costs, we retain the transparency and replicability of investments in canonical Fama-French long-short portfolios at the expense of some realism for a long-only investor.

V. Conclusion

Some firm characteristic trading strategies are relatively straightforward to implement, because of high autocorrelation or large capitalization and liquid long and short positions. We call these “tilts.” We compute the optimal allocation to four tilts over the period from 1968 through 2014. Value, small size, high profits, and low beta all get positive shares; they are 20%, 26%, 23%, and 24%, respectively. This past evidence suggest a simple approach to factor investing for practitioners that gives these canonical tilts roughly an equal weight that increases with tolerance for tracking error. Lower autocorrelation and less liquid characteristic strategies also have a part in stock selection, but their optimal allocations are much more sensitive to portfolio size. Reasonable estimates of transaction costs, which are dependent on assets under management, must be deducted from the average returns, to provide rough estimates of their allocations alongside lower cost tilts.

The large allocation to low beta stands in contrast to recent papers by Novy-Marx (2014) and others, which claim that low risk strategies are subsumed by value or high profits. We find different results with a more predictive measure of beta, with a consistent, long-short portfolio construction that treats the strategies on an apples-to-apples basis, and by separating out tilts from higher frequency “trades.”

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Figure 1. Optimal Tilts, 1968-2014. We compute the mean-variance optimal tilts using in-sample measures of correlation, standard deviation, and annualized return. Tilts in solid bars include: beta, value, size, and profit tilts; and bond market measures of duration and credit. Tilts in hatched bars also include growth. Each strategy is orthogonalized to the overall equity market. See Table VI.

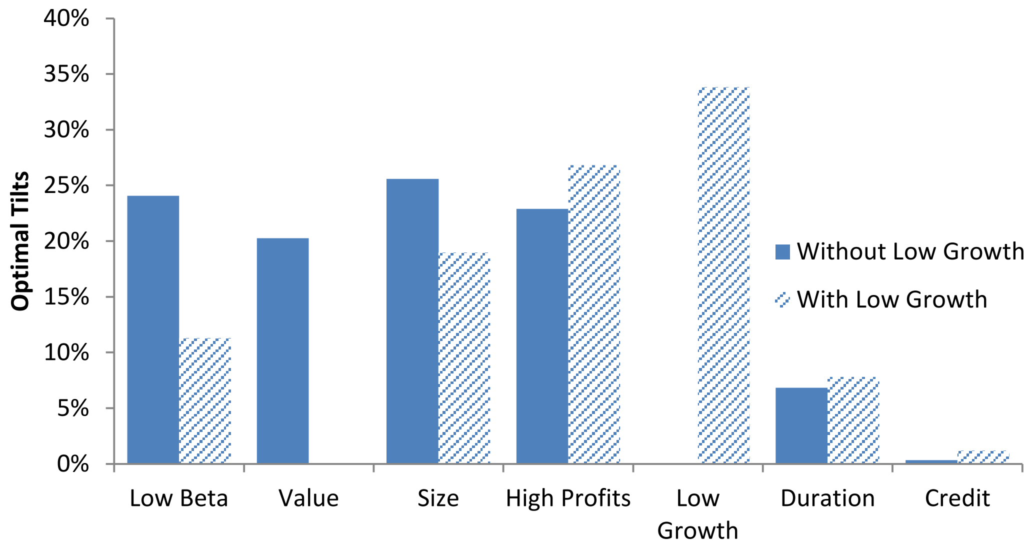

Table I. Sample Characteristic Tilts and Trades.

Tilts	Example Publications
Low Beta 5-Year, 3-Day Overlapping Window Correlation	Black, Jensen, and Scholes, 1972; Baker, Bradley, and Wurgler, 2011; Frazzini and Pedersen, 2014
Low Volatility 1-Year, Daily Volatility	Ang, Hodrick, Xing, and Zhang, 2006 and 2009; Blitz and Van Vliet, 2007
Value Book Equity ÷ Price times Shares Outstanding	Rosenberg, Reid, and Lanstein, 1985; Chan, Hamao, and Lakonishok, 1991; Fama and French, 1992
Small Size Price times Shares Outstanding	Banz, 1981
High Profits (Revenues – COGS – Interest – SG&A) ÷ Book Equity	Basu, 1983; Haugen and Baker, 1996; Cohen, Gompers, and Vuolteenaho, 2002; Fama and French, 2006; Novy-Marx, 2013
Trades	**Example Publications**
Low Growth Assets ÷ Assets, Lagged 1 Year	*Firms with high equity issuance underperform* Ritter, 1991; Loughran and Ritter, 1995; Ikenberry, Lakonishok, and Vermaelen, 1995; Pontiff and Woodgate, 2006; Daniel and Titman, 2006 *Firm with high accruals underperform* Sloan, 1996 *Firms with high asset growth underperform* Hirshleifer, Hou, Teoh, and Zhang, 2004; Cooper, Gulen, and Schill, 2008 *Firms with high investment underperform* Fairfield, Whisenant, and Yohn, 2003; Titman, Wei, and Xie, 2004; Xing, 2008
Momentum Return from 12 Months Ago to One Month Ago	*Stock market winners outperform* Jegadeesh, 1990; Jegadeesh and Titman, 1993
Reversal Return from the Previous Month	*Stock market losers outperform* Jegadeesh, 1990; Jegadeesh and Titman, 1995a, b

Table II. Beta Measures, 1968-2014. We examine the predictive power of three different measures of beta. The first uses up to five years of monthly data, with a minimum of 36 months. The second uses the same time period constraints, but instead uses 3-day overlapping returns. The third uses a hybrid approach, with five years of 3-day overlapping returns to compute correlation and 1 year of daily returns to compute volatility. These are assembled into a beta estimate. The table below shows the realized beta spread for simple Fama-French factor portfolios. Fama-French factor portfolios divide the CRSP sample into six, value-weighted portfolios by characteristic (30, 40, 30) and size (above median, below median) and compute the corresponding factor return as the average of the two high characteristic portfolios minus the average of the two low characteristic portfolios. Statistics are for full-sample post-formation monthly portfolio returns, where portfolios are rebalanced monthly and equally weight constituent stocks.

	High Beta		*Low Beta*		*Difference*	
	Beta	**T-stat**	**Beta**	**T-stat**	**Spread**	**T-stat**
5 Year, Monthly Beta	1.38	[47.56]	0.70	[36.45]	-0.68	[-21.58]
5 Year, Overlapping 3-Day Beta	1.39	[52.50]	0.63	[22.45]	-0.77	[-20.89]
5 Year, Overlapping 3-Day Correlation with 1 Year Daily Volatility	1.41	[50.71]	0.63	[22.07]	-0.78	[-20.25]
1 Year Daily Volatility	1.45	[46.39]	0.71	[33.45]	-0.74	[-20.05]

Table III. Tilts and Trades, 1968-2014. We compute performance for seven strategies, annual autocorrelation of firm characteristic values, and the correlation of firm characteristic values with market capitalization. We examine the Fama-French factors, along with momentum, reversal, and beta. We label the first four, which are high autocorrelation strategies, as "Tilts." We label the next three, which are low autocorrelation strategies with low correlation to market capitalization, as illiquid "Trades." In the first set of three columns, we compute the raw return, standard deviation, and Sharpe ratio of simple Fama-French factor portfolios. Fama-French factor portfolios divide the CRSP sample into six, value-weighted portfolios by characteristic (30, 40, 30) and size (above median, below median) and compute the corresponding factor return as the average of the two high characteristic portfolios minus the average of the two low characteristic portfolios. We compute the beta portfolio using 5-year, overlapping 3-day autocorrelations and 1-year daily volatility as in Table I. The remaining portfolios are drawn directly from Ken French's data library. In the second set of four columns, we remove the in-sample beta risk from each portfolio, reporting the beta, alpha, standard deviation, and the Sharpe ratio of the alpha portfolio. In the last set of three columns, we use the definitions in Table I and from Ken French's data library to compute the annual autocorrelation of firm characteristics and the correlation of above-median characteristics and (the opposite of) below-median characteristic with market capitalization from CRSP. Statistics are for full-sample, post-formation monthly portfolio returns, where portfolios are rebalanced annually at the end of each June except for Momentum and Reversal which are rebalanced monthly.

	Simple Strategy Performance, Annualized			*Market-Neutral Strategy Performance, Annualized*				*Characteristic Correlation*			
	Return	**SD**	**Sharpe**	**CAPM Beta**	**CAPM Alpha**	**SD (Alpha)**	**Sharpe (Alpha)**	**1-Year Lag**	**Market Cap +**	**Market Cap -**	**Low Beta**
Tilts											
Low Beta	1.82	16.18	0.11	-0.78	6.43	10.33	0.62	0.88	-0.30	-0.01	1.00
Value	4.51	10.21	0.44	-0.20	5.70	9.68	0.59	0.81	-0.30	-0.04	0.15
Small Size	2.41	10.71	0.22	0.19	1.31	10.29	0.13	0.97	-1.00	1.00	0.26
High Profits	3.19	7.62	0.42	-0.10	3.79	7.45	0.51	0.72	-0.01	-0.34	-0.01
Trades											
Low Growth	4.48	6.93	0.65	-0.18	5.52	6.34	0.87	0.30	-0.33	-0.05	0.02
Momentum	7.89	15.04	0.52	-0.13	8.64	14.91	0.58	0.05	-0.31	-0.06	-0.08
Reversal	5.86	11.21	0.52	0.21	4.65	10.72	0.43	0.03	-0.12	-0.21	0.02

Table IV. Shared Risk in Beta Tilts, 1968-2014. We decompose the variance of a long-short beta portfolio into components shared by: value, size, and profit tilts; growth, momentum, and reversal trades; and bond market measures of duration and credit. Each strategy is orthogonalized to the overall equity market. The remaining variance (last column) is unique to a beta tilt. Statistics are for full-sample, post-formation monthly portfolio returns, where portfolios are rebalanced annually at the end of each June except for momentum and reversal which are rebalanced monthly.

	Market-Neutral Covariances										
	Value	**Small Size**	**High Profits**	**Low Growth**	**Mom**	**Re-versal**	**Duration**	**Credit**	**Multi Variate**	**Over-lap**	**Unique**
Tilts											
Value	0.31								0.20		
	[4.25]								[2.30]		
Small Size		-0.41							-0.29		
		[-7.28]							[-6.54]		
High Profits			0.48						0.33		
			[6.17]						[5.39]		
Trades											
Low Growth				0.38					0.27		
				[4.25]					[2.86]		
Momentum					0.13				0.15		
					[2.14]				[3.55]		
Reversal						-0.07			-0.01		
						[-0.78]			[-0.17]		
Credit Market Controls											
Duration							0.31		0.25		
							[6.33]		[4.64]		
Credit								-0.22	0.17		
								[-1.82]	[1.65]		
Variance Explained	9.10	18.09	12.96	5.95	3.99	0.56	10.58	1.31	43.50	-19.04	63.30
Percent Variance Explained (%)	8.52	16.94	12.13	5.57	3.74	0.52	9.91	1.22	40.73	-17.83	59.27

Table V. Sources of Return in Beta, 1968-2014. We decompose the return on a long-short beta portfolio into components shared by: value, size, and profit tilts, and by bond market measures of duration and credit. Each strategy is orthogonalized to the overall equity market. The remaining return is unique to a beta tilt. Statistics are for full-sample, post-formation monthly portfolio returns, where portfolios are rebalanced annually at the end of each June.

		Market-Neutral Explained Returns, Annualized							
	Market Model	**Value**	**Small Size**	**High Profits**	**Duration**	**Credit**	**All**	**Overlap**	**Unique**
Annualized Alpha (%)	6.43	5.70	1.31	3.79	3.01	-0.40			
	[4.24]								
Tilts									
Value		0.31					0.27		
		[4.25]					[4.44]		
Small Size			-0.41				-0.29		
			[-7.28]				[-6.24]		
High Profits				0.48			0.30		
				[6.17]			[4.89]		
Credit Market Controls									
Duration					0.31		0.25		
					[6.33]		[4.37]		
Credit						-0.22	0.07		
						[-1.82]	[0.55]		
Alpha Explained (%)		1.78	-0.54	1.83	0.92	0.09	3.02		3.41
Percent Alpha Explained (%)		27.64	-8.40	28.50	14.35	1.38	46.98	-16.48	53.02

Table VI. Optimal Shares, 1968-2014. We compute the mean-variance optimal shares, using in-sample measures of correlation, standard deviation, and annualized return. Tilts include: beta, value, size, and profit tilts; and bond market measures of duration and credit. Each strategy is orthogonalized to the overall equity market. Statistics are for full-sample, post-formation monthly portfolio returns, where portfolios are rebalanced annually at the end of each June.

		Market-Neutral Strategy Performance, Annualized			*In Sample Correlations*					
	Optimal Share	**Raw (%)**	**Per 1-Month SD (%)**	**Annual Sharpe**	**Beta**	**Value**	**Small Size**	**High Profits**	**Duration**	**Credit**
Tilts										
Low Beta	24%	6.43	2.16	0.62	1.00					
Value	20%	5.70	2.04	0.59	0.29	1.00				
Small Size	26%	1.31	0.44	0.13	-0.41	-0.04	1.00			
High Profits	23%	3.79	1.76	0.51	0.35	0.04	-0.34	1.00		
Credit Market Controls										
Duration	7%	3.01	0.98	0.28	0.31	0.05	-0.16	0.10	1.00	
Credit	0%	-0.40	-0.27	-0.08	-0.11	0.11	0.09	-0.11	-0.49	1.00
Portfolio (Annual)			3.33	0.96						

Table VII. Tilted Portfolios. We use the optimal shares computed in Table VI to produce illustrative portfolios at active risks ranging from 0.5% to 10%. We compute these using a starting point of 60% equity at market capitalization weights and 40% cash. We start with the weights on the unit standard deviation portfolios orthogonalized to the market to derive weights on standard Fama and French high minus low portfolios that are not orthogonalized, credit markets, the value-weighted market, and cash.

				Example Portfolio Tilt for Annual Active Risk Of:					
	Optimal 1-SD Share	**SD**	**Beta**	**0%**	**0.5%**	**1%**	**2%**	**5%**	**10%**
Tilts									
Low Beta	24%	2.98	-0.78	0.0%	2.6%	5.1%	10.2%	25.6%	51.2%
Value	20%	2.80	-0.20	0.0%	2.3%	4.6%	9.2%	23.0%	46.0%
Size	26%	2.97	0.19	0.0%	2.7%	5.5%	10.9%	27.3%	54.7%
High Profits	23%	2.15	-0.10	0.0%	3.4%	6.8%	13.5%	33.8%	67.6%
Credit Market Controls									
Duration	7%	3.06	0.09	0.0%	0.7%	1.4%	2.8%	7.1%	14.1%
Credit	0%	1.47	0.07	0.0%	0.1%	0.1%	0.3%	0.7%	1.5%
Market				60.0%	62.2%	64.5%	69.0%	82.4%	104.8%
Cash				40.0%	37.8%	35.5%	31.0%	17.6%	-4.8%
Incremental Return				0.0%	0.5%	1.0%	1.9%	4.8%	9.6%