Leverage and the Beta Anomaly

Malcolm Baker, Mathias F. Hoeyer, and Jeffrey Wurgler [ID]*

Abstract

The well-known weak empirical relationship between beta risk and the cost of equity (the beta anomaly) generates a simple tradeoff theory: As firms lever up, the overall cost of capital falls as leverage increases equity beta, but as debt becomes riskier the marginal benefit of increasing equity beta declines. As a simple theoretical framework predicts, we find that leverage is inversely related to asset beta, including upside asset beta, which is hard to explain by the traditional leverage tradeoff with financial distress that emphasizes downside risk. The results are robust to a variety of specification choices and control variables.

I. Introduction

Millions of students have been taught corporate finance under the assumption of the capital asset pricing model (CAPM) and integrated equity and debt markets. Yet, it is well known that the link between textbook measures of risk and realized returns in the stock market is weak, or even backward. For example, a dollar invested in a low beta portfolio of U.S. stocks in 1968 grows to $70.50 by 2011, while a dollar in a high beta portfolio grows to just $7.61 (see Baker, Bradley, and Taliaferro (2014)). The evidence on the anomalously low returns to high-beta stocks begins as early as Black, Jensen, and Scholes (1972). In a recent contribution, Bali, Brown, Murray, and Tang (2017) identify it as "one of the most persistent and widely studied anomalies in empirical research of security returns" (p. 2369). A large literature has come to view the beta anomaly, and related anomalies based on total or idiosyncratic risk, as evidence of mispricing as opposed to a misspecified risk model. In this paper, we consider these

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explanations sufficiently plausible to consider their implications for leverage. In doing so, we bring ideas from the anomalies literature into corporate finance, which often assumes efficient and integrated securities markets to highlight other frictions.

We show that a beta anomaly in equity markets, with the absence of any other deviations from the Modigliani–Miller setup, leads to a simple tradeoff. Intuitively, under the anomaly, beta risk is overvalued in equity securities, but not in debt securities. Ideally, then, to minimize the cost of capital, risk is concentrated in equity. A firm will always want to issue as much riskless debt as it can. This lowers the cost of equity by increasing its beta without any "inefficient" transfer of risk from equity to debt. But, as debt becomes risky, further increases in leverage have a cost. Shifting overvalued risk in equity securities to fairly valued risk in debt securities increases the overall cost of capital. For firms with high-beta assets, this increase is high even at low levels of leverage. For firms with low-beta assets, this increase remains low until leverage is high. Using the Merton (1974) model to characterize the functional form of debt betas and the underlying transfer of risk from equity to debt, we show that there is an interior optimum leverage ratio that is inversely related to asset beta.

The theory's immediate prediction is that leverage is negatively related to asset beta. Empirically, we confirm that there is indeed a robust negative relationship between leverage and asset beta. The relationship remains strong when controlling for overall asset variance (a control for distress risk), using alternative measures of leverage and industry measures of asset beta and overall risk, and including various other control variables.

While it is reassuring that the beta anomaly tradeoff generates a leverage-asset beta prediction that is borne out in the data, an obvious alternative explanation is based on the standard tradeoff theory, which also predicts an inverse relationship between leverage and risk (most intuitively, total asset risk, not asset beta). For example, Long and Malitz (1985) include systematic asset risk, not just overall risk, in leverage regressions with the traditional tradeoff between taxes and financial distress costs in mind. We do not wish to cast doubt on the relevance of the standard tradeoff theory. Instead, we derive and test additional predictions of the beta anomaly tradeoff to show that it has incremental explanatory power.

In particular, we prove that, under a beta anomaly, both "upside" and "downside" betas are inversely related to leverage. Financial distress costs, on the other hand, clearly emphasize downside risk. Moreover, consistent with the beta anomaly tradeoff, we find that leverage is indeed inversely related to upside asset beta as well. In some specifications, the upside beta relationship is actually stronger than the downside beta relationship. While our results do not rule out the traditional tradeoff theory, given that downside beta still matters, this prediction and empirical relationship is most easily explained by a beta anomaly tradeoff.

We tentatively suggest that the beta anomaly tradeoff may also help to explain extremely high and low leverage levels that challenge the standard tradeoff. For instance, Graham (2000) and others have pointed out that hundreds of profitable firms, with high marginal tax rates, maintain essentially zero leverage. This is often called the low-leverage puzzle. Conversely, a number of other profitable firms maintain high leverage despite no tax benefit. The beta anomaly tradeoff

could help to explain these patterns. If low leverage firms have determined that the tax benefit of debt is less than the opportunity cost of transferring risk to lower-cost equity, low leverage may be optimal even in the presence of additional frictions; a minor, realistic transaction cost of issuance could drive some firms to zero leverage. Meanwhile, low asset beta firms with no tax benefits of debt still resist equity because of its high risk-adjusted cost at low levels of leverage, and instead use a very large fraction of debt finance.

In addition to providing a novel theory of leverage based on a single deviation from the Modigliani–Miller setup, the beta anomaly approach has an attractive and unifying conceptual feature. The traditional tradeoff theory, under rational asset pricing, cannot fit both the leverage and asset pricing evidence on the pricing of beta. If beta *is* truly a measure of risk relevant for capital structure, then presumably it would also help to explain the cross section of asset returns, which it does not. Investors, recognizing the associated investment opportunities, would demand higher returns on assets exposed to the systematic risk of fire sales, with high risk-adjusted costs of financial distress. If beta *is not* a measure of risk, as the large literature that follows Fama and French (1992), (1993) has claimed, then asset beta should not be a constraint on leverage, after controlling for total asset risk. Although the beta anomaly is far from the only force at work in real-life capital markets (following the standard approach in corporate finance theory, we focus on a single "friction" (the beta anomaly) for simplicity), it is worth noting that it offers an internally consistent explanation for both of these patterns.

Section I briefly reviews the beta anomaly and derives optimal leverage under the anomaly. Section II contains empirical hypotheses and tests. Section III concludes.

II. The Beta Anomaly Tradeoff

A. Motivation and Setup

Over the long run, riskier asset *classes* have earned higher returns, but the historical risk-return tradeoff *within* the stock market is flat at best. The CAPM predicts that the expected return on a security is proportional to its beta, but in fact stocks with higher beta have tended to earn lower returns, particularly on a risk-adjusted basis. As mentioned earlier, Bali et al. (2017) highlight the beta anomaly as one of the most *prominent* of the many dozen anomalies studied in the literature. We refer the reader to that paper, as well as Baker, Bradley, and Wurgler (2011) or Hong and Sraer (2016) for an overview of the large literature that shows the beta anomaly's robustness across sample periods and international markets and reviews some of its theoretical foundations.

The simple linear specification for the beta anomaly that we employ here is

$$(1) \qquad r_{\epsilon} \quad = \quad (\beta_{\epsilon} - 1)\gamma + r_f + \beta_{\epsilon} r_p,$$

where r_f is the risk-free rate, r_p is the market risk premium, and $\gamma < 0$ measures the size of the anomaly. The beta anomaly is that a stock has "alpha" and the alpha falls with its beta.

A few comments on this simple functional form. In terms of the "neutral point" where equity risk is fairly priced, equation (1) assumes this takes place at an

equity beta of 1.0. The equity premium puzzle suggests that, historically, location might be at a higher level. But, even if equity were undervalued or overvalued on average, the mean leverage behavior would change, but predictions about the cross section of leverage, our focus here, would not.

If the beta anomaly extended in equal force into debt, Modigliani–Miller irrelevance would still hold. Neither the literature nor our own tests indicate a meaningful beta anomaly in debt, however. In Frazzini and Pedersen (2014), short-maturity corporate bonds of low risk firms have marginally higher bond-market-beta risk-adjusted returns, but bond index betas have little relationship to stock index betas, the basis of the beta anomaly. In fact, Fama and French (1993) show that stock market betas are nearly identical for bond portfolios of various ratings, and Baele, Bekaert, and Inghelbrecht (2010) find that even the sign of the correlation/beta between government bond and stock indexes is unstable. Nonetheless, just to be sure, we directly compared the returns on beta-sorted stock portfolios with beta-risk-adjusted corporate bond portfolios and can easily reject an integrated beta anomaly. These results are omitted for brevity but available on request. See Harford, Martos-Vila, and Rhodes-Kropf (2015) for perspectives on financing under (non-beta-based) debt mispricing.

With a beta anomaly in equity, the overall cost of capital depends not only on asset beta but on leverage:

$$
\begin{aligned}
(2) \qquad \mathrm{WACC}(e) &= er_\epsilon + (1-e)r_d \\
&= r_f + \beta_a r_p + (\beta_a - 1)y - (1-e)(\beta_d - 1)y,
\end{aligned}
$$

where β_a is asset beta and e is capital structure as measured by the ratio of equity to firm value. The second to last term (the asset beta minus 1 times γ) is the uncontrollable reduction in the cost of capital that comes from having high-risk assets. The last term is the controllable cost of having too little leverage while debt beta remains low.

An advantage of using the CAPM to develop comparative statics is to see the familiar textbook transfers of beta risk from equity to debt as leverage increases. However, any asset pricing model that features a stronger beta anomaly in equity will lead to the same qualitative conclusions.

B.   Optimal Capital Structure

Next we outline a simple, static model of optimal capital structure with no frictions other than a beta anomaly. There are no taxes, transaction costs, issuance costs, incentive or information effects of leverage, or costs of financial distress. It is interesting that unlike other tradeoff approaches, which require one friction to limit leverage on the low side and another to limit it on the high side, this single mechanism can drive an interior optimum.

The optimal capital structure minimizes the last term of equation (2) by satisfying the first-order condition for e. With the further assumption of a differentiable debt beta, for a given level of asset beta the optimal capital ratio e^* satisfies

$$
(3) \qquad -y\left\{1 - \beta_d[e^*(\beta_a)] + [1 - e^*(\beta_a)]\frac{\partial \beta_d[e^*(\beta_a), \beta_a]}{\partial e}\right\} = 0,
$$

or, in terms of optimal debt beta,

$$\beta_d^*[e^*(\beta_a), \beta_a] \quad = \quad 1 + [1 - e^*(\beta_a)]\frac{\partial \beta_d[e^*(\beta_a), \beta_a]}{\partial e}.$$

The optimum leverage does not depend on the size of the beta anomaly, but this is a bit of a technicality. If there were any other frictions associated with leverage, such as taxes or financial distress costs, the anomaly's size would become relevant.

Observation 1. Firms will issue some debt, and as much debt as possible as long as it remains risk-free.

The first-order condition cannot be satisfied as long as the debt beta is 0. At a zero debt beta, the left side of equation (3) is positive (recall $\gamma < 0$). In other words, issuing more equity at the margin will raise the cost of capital. With zero debt, the asset beta is equal to the equity beta and the WACC reduces to:

(4) $$\text{WACC}(1) \quad = \quad (\beta_a - 1)y + r_y + \beta_a r_p.$$

A first-order Taylor approximation around $e = 1$ shows that even marginal debt will decrease the cost of capital:

(5) $$\text{WACC}(e) \quad \approx \quad \text{WACC}(1) + (1 - e)\lambda \quad < \quad \text{WACC}(1).$$

At first blush, this would seem to deepen the low leverage puzzle of Graham (2000). One might ask why nonfinancial firms do not increase their leverage ratios further to take advantage of the beta anomaly: It is initially unclear how the low leverage ratios of nonfinancial firms represent an optimal tradeoff between the tax benefits of interest and the costs of financial distress, much less an extra benefit of debt arising from the mispricing of low risk stocks.

The answer from equation (3) is that many low leverage firms (e.g., the stereotypical unprofitable technology firm) already start with a high asset beta or overall asset risk. Their assets are already quite risky at zero debt. Even at modest levels of debt, meaningful risk starts to be transferred to debt. While equation (3) cannot on its own explain why a firm would have exactly zero debt, it can explain why some firms have low levels of debt, despite the tax benefits and modest costs of financial distress.

Observation 2. Leverage has an interior optimum.

Zero equity is also not optimal. With all debt finance, the debt beta equals the asset beta and equation (2) reduces to the traditional WACC formula without the beta anomaly. This establishes that optimum leverage must be interior. The intuition is that, with the assumption of fairly priced debt, the firm will be fairly priced if it is funded entirely with debt (i.e. 100% leveraged). Can it increase value by reducing its leverage ratio? Yes. This new equity, an out-of-the-money call option, will be high risk, and hence overvalued. As a consequence, neither 0% nor 100% leverage are optimal, so there must be an interior optimum.

To further our understanding of optimal debt levels, we must characterize the dynamics underlying the transfer of risk from equity to debt with increasing levels of leverage, and in particular the dependence of debt beta on leverage.

A natural candidate for the functional form of debt betas is the Merton (1974) model. Merton uses the isomorphic relationship between levered equity, a European call option, and the accounting identity $D = V - E$ to derive the value of a single, homogeneous debt claim, such that

$$(6) \qquad D(d,T) \quad = \quad Be^{-r_f t}\Phi[x_2(d,T)] + V\{1 - \Phi[x_1(d,T)]\},$$

where V is firm value with volatility σ, D is the value of the debt with maturity in τ and face value B. Let $T = \sigma^2\tau$ be the firm variance over time, and $d \equiv \frac{Be^{-r_f \tau}}{V}$ the debt ratio, where debt is valued at the risk-free rate, thus d is an upward biased estimate of the actual market based debt ratio (Merton (1974), pp. 454–455). Here, $\Phi(\mathbf{x})$ is the cumulative standard normal distribution and x_1 and x_2 are the familiar terms from the Black–Scholes formula.

Following the approach of Black and Scholes (1973), we arrive at the debt beta:

$$(7) \qquad \beta_d \quad = \quad \beta_a \frac{V}{D} D_V.$$

Here D_V is the first derivative of the debt value given in equation (6) with respect to V. In the Merton model, debt is equivalent to a riskless debt claim less a put option. It follows that the derivative D_V is equivalent to the negative of the derivative of the value of this put option. That is, the derivative (or delta) of the put option on the underlying firm value is $\Delta_{put} = -[1 - \Phi(x_1)]$, thus $D_v \equiv -\Delta_{put} = 1 - \Phi(x_1)$. Substituting for equation (2), the debt beta can be written as

$$(8) \qquad \beta_d \quad = \quad X(d)\beta_a,$$

where $X(d) = \frac{1 - \Phi(x_1)}{d\Phi(x_2) + 1 - \Phi(x_1)}$ is positive. Further, we have that the debt beta is bounded:

$$\lim_{d \to 0}\{\beta_d = X(d)\beta_a\} \quad = \quad 0, \quad \text{and} \quad \lim_{d \to \infty}\{\beta_d = X(d)\beta_a\} \quad = \quad \beta_a$$

in line with the boundary conditions of the debt and in support of the limiting conditions necessary to establish the claim of an interior optimum leverage.

The factor $\Phi(x_1)$ is equivalent to the delta of an equity claim with spot price equal to V and exercise price equal to face value of the debt B. In light thereof, the debt beta can be seen to be driven by the increasing value loss in default. If $\beta_a > 0$ then the debt beta will be continuous and strictly increasing in d. Now rewriting equation (8),

$$(9) \qquad \beta_d \quad = \quad \beta_a \frac{V - V\Phi(x_1)}{D},$$

and following the limits above it can be seen that $0 \le V - V\Phi(x_1) \le D$. Consequently, $V\Phi(x_1)$ in equation (9) can be interpreted as the conditional expectation of firm value given it is larger than the face value of debt, times the probability of the firm value being larger than the face value of debt. This is effectively the amount of firm risk carried by the debt.

On closer inspection, the debt beta in equation (8) can be written, showing its full functional dependence, $\beta_d(d, \beta_a, T)$. In our framework, however, the measure of leverage is not d, but rather the market-based capital ratio, e, that is given by

$$(10) \qquad e(d,T) \quad = \quad \frac{V-D}{V} \quad = \quad \Phi(x_1) - d\Phi(x_2),$$

and is continuous and strictly decreasing in d.

By expressing the debt beta in equation (8) parametrically as a function of the equity ratio in equation (10) with d as a shared parameter, holding all else equal, the derivative of the debt beta in equation (3) is equivalent to:

$$(11) \qquad \frac{\partial\beta_d(e(d,T), \beta_a, T)}{\partial e(d,T)} \quad = \quad \frac{\partial\beta_d(d, \beta_a, T)/\partial d}{\partial e(d,T)/\partial d},$$

and

$$\frac{\partial\beta_d(d, \beta_a, T)/\partial d}{\partial e(d,T)/\partial d} \quad = \quad -M(d)\beta_a.$$

The intuition of $M(d)$ is not important. The first feature of $M(d)$ is that it does not depend on the asset beta β_a itself. So, we can separate out the transfer of risk (which depends on d and the distribution of firm values characterized by the inputs of total risk, time to maturity, the riskless rate, and the face value of debt) from the specific risk transferred:

$$(12) \quad M(d) \quad = \quad \frac{\Phi(x_1)\Phi(x_2) + \Phi(x_2)\dfrac{\phi(x_1)}{\sigma\sqrt{\tau}} + [1 - \Phi(x_1)]\dfrac{\phi(x_2)}{\sigma\sqrt{\tau}} - \Phi(x_2)}{[1 - \Phi(x_1) + d\Phi(x_2)]^2 \left[\Phi(x_2) - \dfrac{\phi(x_2)}{\sigma\sqrt{\tau}} + \dfrac{\phi(x_1)}{d\sigma\sqrt{\tau}}\right]}.$$

A second feature of M is that it is positive, so the debt beta falls as the capital ratio increases.

It is worth mentioning that a cost of equity anomaly in which average returns vary but risk does not (e.g., Internet or blockchain assets are overvalued, or more generally the type of mispricing described in Baker and Wurgler (2002)) does not lead to an interior optimum leverage on its own. Without another friction and without the issuance itself changing valuations, such as downward sloping demand, one gets a corner solution of all equity if equity is overvalued, or all debt if equity is undervalued.

Stein (1996) solves this problem by adding costs of financial distress. A mispricing in which risk varies but average returns do not, such as the beta anomaly, is fundamentally different because the act of levering or delevering itself changes the risk of the equity, and hence how much it is mispriced. As just shown, overvalued firms may now want to issue a small amount of debt, and undervalued firms will not fund themselves with 100% debt because as leverage rises, their equity is no longer undervalued. This mechanism is not present in Stein, and allows us to derive an interior optimum based on a beta anomaly alone.

Observation 3. The optimal leverage ratio is decreasing in asset beta.

This is a basic observation and an intuitive, testable prediction. With the Merton assumptions, and with no frictions other than a beta anomaly in equity, the optimal level of capital, e^* in equation (3), can be expressed with equations (8) and (11) as

$$(13) \qquad \begin{aligned} e^*(d^*, \beta_a) &= 1 - \frac{1}{M(d^*)} \left(\frac{1 - \beta_d(d^*, \beta_a)}{\beta_a} \right) \\ &= 1 - \frac{1}{M(d^*)} \left(\frac{1}{\beta_a} - X(d^*) \right). \end{aligned}$$

Holding the positive functions X and M constant (we are varying the asset beta without changing the overall level of risk) the optimal level of capital is increasing in asset beta:

$$(14) \qquad \frac{\partial e^*(d^*, \beta_a)}{\partial \beta_a} = \frac{1}{M(d^*)\beta_a^2} \geq 0.$$

We outlined the intuition in the introduction. Under the beta anomaly, beta is overvalued in equity securities. Ideally, to minimize the cost of capital, risk is concentrated in equity. This led to the first observation that firms will issue as much risk-free debt as possible. This lowers the WACC through an increase in the cost of equity due to its increased beta. Once debt becomes risky, further increases in leverage, or reductions in equity capital e, have a cost. At that point, M is greater than 0. Shifting overvalued risk in equity to fairly valued risk in debt increases the cost of capital. For firms with high-beta assets, this increase is high even at low levels of leverage. For firms with low-beta assets, this increase remains low until leverage is high.

More formally, we can show that:

$$(15) \qquad \begin{aligned} \frac{\partial \beta_d(e, \beta_a, T)}{\partial e} &< 0, \\ \frac{\partial \beta_d(e, \beta_a, T)}{\partial \beta_a} &\geq 0, \\ \frac{\partial^2 \beta_d(e, \beta_a, T)}{\partial \beta_a \partial e} &< 0. \end{aligned}$$

The first 2 partial derivatives of the debt beta with respect to the equity ratio and the asset beta follow directly from equation (9) and the assumption that $\beta_a > 0$. Furthermore, since the asset beta is simply a positive scalar, the cross partial derivative with the equity ratio must have the same sign as the partial derivative with respect to the equity ratio.

We can now sign the change in the optimal capital ratio as a function of the underlying asset beta in the more general case. Taking the derivative of e^* with respect to asset beta:

$$(16) \qquad \begin{aligned} \frac{de^*(\beta_a)}{d\beta_a} = &- \left[\frac{\partial \beta_d(\beta_a)}{\partial \beta_a} + [1 - e^*(\beta_a)] \frac{\partial^2(e^*, \beta_a)}{\partial e \partial \beta_a} \right] \\ &\times \left[-2 \frac{\partial \beta_d(e^*(\beta_a), \beta_a)}{\partial e} + [1 - e^*(\beta_a)] \frac{\partial^2 \beta_d(e^*(\beta_a), \beta_a)}{\partial e^2} \right]. \end{aligned}$$

The first term is positive using the signs of the partial derivatives in equation (15). The second term is the second order condition at the optimal leverage ratio defined in equation (3). It follows from Cargo (1965) that the second order condition is positive, if and only if both the debt beta in equation (8) and 1 minus the capital ratio in equation (10) are strictly increasing in d and equation (11) is negative, which we have already established.

Observation 4. Upside and downside asset beta are equally important in determining optimal leverage.

Although the mechanism is very different, the predictions so far are intuitively consistent with the standard tradeoff theory and associated evidence. This is encouraging but it is also important to derive a testable prediction that allows us to empirically distinguish the beta anomaly tradeoff from the costs of financial distress and the traditional tradeoff. To do this, we make the simple observation that in the traditional tradeoff, only downside beta, not upside beta, increases the costs of financial distress and therefore reduces optimal leverage. In contrast, in the beta anomaly tradeoff, the effect is symmetric. To show this formally, we repeat the previous exercise with a version of equation (1) that separates beta into upside (β^+) and downside (β^-) components:

$$(17) \qquad r_e \quad = \quad \left(\beta^+ + \beta^- - 1\right)\gamma + r_f + (\beta^+ + \beta^-)r_p,$$

which we define in the semivariance spirit of Markowitz (1959) and Hogan and Warren (1974).[1]

We can now substitute this decomposition of the asset beta into (13),

$$(18) \qquad \beta_a \quad = \quad \beta_a^+ + \beta_a^-,$$

which implies that the optimal capital structure takes the following form:

$$(19) \qquad e^*(d^*, \beta_a) \quad = \quad 1 - \frac{1}{M(d^*)}\left(\frac{1}{\beta_a^+ + \beta_a^-} - X(d^*)\right).$$

This means that, once again holding the positive functions X and M constant, the optimal level of capital is rising to the same degree in both upside and downside asset beta, as in equation (14).

Again, there is a straightforward intuition for equation (19). The character of total risk, as defined in the functions M and X, is what dictates the transfer of risk from equity to debt. Holding this transfer of risk constant, the optimal level of capital is increasing in any measure of mispriced asset risk, whether upside or downside.

Observation 5. A negative empirical relationship between equity beta and leverage is sufficient to prove a negative empirical relationship between asset beta and leverage.

The initial step in our empirical analysis is to confirm a negative relationship between asset beta and leverage. However, to measure asset beta directly, all

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[1] $\beta \equiv \frac{1}{var(r_m)}n^{-1}[\sum_{j=1}^{n}(r_{i,j}-\mu_i)(r_{m,j}-\mu_m)I_{r_{m,j>T}} + \sum_{j=1}^{n}(r_{i,j}-\mu_i)(r_{m,j}-\mu_m)I_{r_{m,j\leq T}}]=\beta^+ + \beta^-.$

liabilities must be traded, which is rarely the case, so the traditional method of estimating asset beta is simply to assume that debt betas are 0. Observation 5 is housekeeping that allows us to avoid having to condition a negative leverage-asset beta link on a zero debt beta assumption. If leverage rises as equity beta falls, then asset beta must fall too through a simple Modigliani–Miller logic and absolute priority of returns. If we find that the equity beta is lower and leverage is higher, then the asset beta must be lower. Thus, a negative relationship between equity beta and leverage further implies a negative relationship between asset beta and leverage.

Further Remarks

Like all theoretical work on leverage, our framework clearly makes tradeoffs between tractability and realism. But, one broader question is what do managers really need to know, in practice, in order to drive the predictions made above? In the model, managers must understand how their leverage choice affects their cost of capital at the margin. In "reality," a reasonable suggestion is that managers estimate their cost of equity capital in the process of estimating the fundamental value of their common stock. The manager's valuation depends, as in textbooks, on his or her best guess of future cash flows and the firm's true, rational cost of capital.

In such calculations, according to surveys by Graham and Harvey (2001), "the CAPM is by far the most popular method of estimating the cost of equity capital: 73.5% of respondents always or almost always use the CAPM." So, these calculations amount to an estimate of the cost of capital by CFOs. When the firm is undervalued according to the CAPM, and their cost of capital is higher than it ought to be, these firms are presumably less likely to issue equity and more likely to rely on debt as a marginal source of finance.

The essential result for our purpose is that if the stock market contains a beta anomaly in which the security market line is empirically too flat, firms with low asset betas choose higher leverage. This goes through with a weaker assumption that managers are assessing the value of their firm's common equity with the CAPM, as Graham and Harvey (2001) say that they do, as opposed to having full knowledge of the return generating process and their influence on it.

We might also mention a possible extension to the case of three types of capital, such as investment grade debt, subordinated or junk debt, and equity, in which a beta or risk anomaly is present in both investment grade debt and equity, but not in junk debt. What would be challenging about such an extension is the computation of the endogenous, maximum quantity of pari passu investment grade debt, as a function of asset volatility. We conjecture that for many firms there would be a corner solution at the investment grade boundary (i.e. at the maximum amount of investment grade debt and zero junk debt). We save such an extension for future work.

C.   A Calibration

Here we assume the model is correct, including its absence of other frictions, to make some coarse calculations about the value of exploiting the beta anomaly. To keep things simple, we use the Black–Scholes assumptions and a single

liquidation date, 5 years forward, with a contractual allocation of value between debt and equity, and no costs of financial distress. For each level of leverage, we compute the value of debt, the value of equity, and the equity beta using the Merton model. Although this approach is a far cry from structural estimations of dynamic capital structure models, it may provide a degree of insight into the potential empirical relevance of the beta anomaly for leverage.

Figure 1 shows the cost of capital and firm value as a function of leverage for a variety of asset risk levels. In the absence of a beta anomaly, cash flows both grow and are discounted at the CAPM rate, so firm value is the same, at $10 under our assumptions, at all risk levels. In the figure we modify the value of equity using the beta anomaly in equation (1) of $\gamma = -5\%$ per year. This is conservative; in Baker et al. (2011) sample of U.S. stocks between 1968 and 2008, the difference in alphas between the top and bottom beta quintile portfolios is -7.5% per year.

Figure 1 shows how a high equity beta uses the anomaly to raise value, while a low equity beta reduces value in passing it up. Because the only effects here are

FIGURE 1

Value Effects of Leverage When There Is a Beta Anomaly in Equities

In Figure 1, we compute firm value for firms with 5 levels of asset beta. Each firm has a normally distributed terminal value of 5 years, hence, with a contractual distribution of value between debt and equity and no costs of financial distress or tax effects. The value of each firm would be exactly $10, regardless of leverage, if there were no low-beta anomaly. Volatility is equal to asset beta times the sum of a market volatility of 16% plus an idiosyncratic firm volatility of 20%. The risk-free rate is 2%. We compute the value of equity, the value of debt, and the equity beta under the Merton model with no beta anomaly. We compound this equity value using the CAPM expected return with a market risk premium of 8% over 5 years. So, a firm with a beta of 0.25 (beta of 2.0) has a weighted average cost of capital of 4% (18%) in the absence of a beta anomaly. We then present value this future equity value using the discount rate from equation (1) with a γ of -5%. This is the adjusted equity value. The weighted average cost of capital uses the adjusted equity value and the value of debt as weights, the cost of equity from equation (1), and the CAPM for debt. Firm value is the adjusted equity value plus the value of debt. Leverage is computed using these market values.
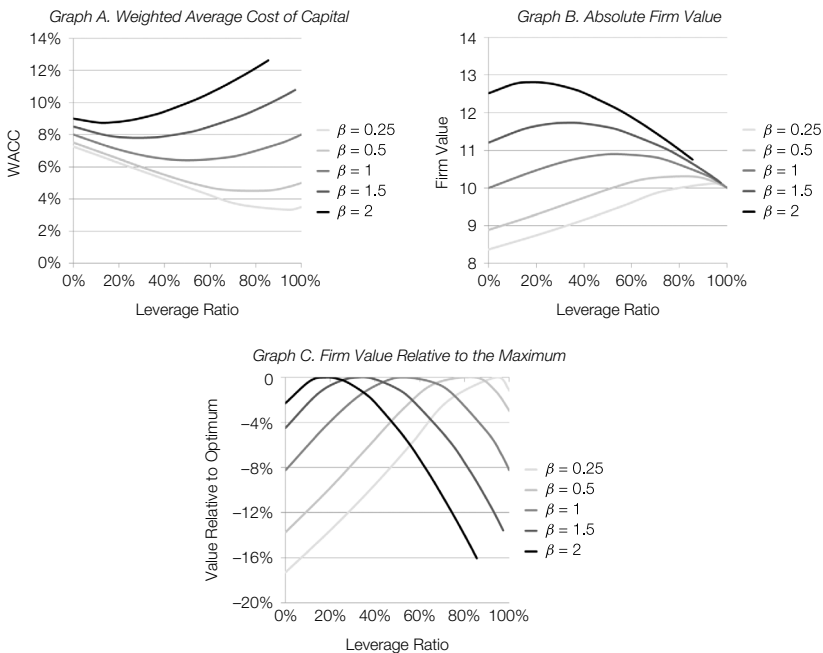
through the weighted average cost of capital, with no cash flow effects, a weighted average cost of capital minimum in Graph A is equivalent to a firm value maximum in Graph B. Finally, under a beta anomaly, high asset beta means higher valuations at any level of leverage. Graph C removes this effect and shows value relative to the maximum for each level of asset beta. Again, like any calibration exercise, the conclusions are conditional on the validity of the underlying assumptions. What this exercise suggests is that under a stylized model with reasonable parameters, not exploiting the anomaly can substantially reduce firm value.

III.   Empirical Tests

A.   Hypotheses

The observations above lead to three hypotheses associated with a beta anomaly tradeoff. The first is basic, the second separates the beta anomaly tradeoff from a traditional explanation, and the third provides an aspect of robustness.

Hypothesis 1. Leverage is negatively related to asset beta.

This hypothesis is motivated by Observation 3. Support for this hypothesis is best viewed as an initial, necessary condition for the beta anomaly tradeoff to be empirically relevant, in the sense that it cannot on its own rule out alternative hypotheses based on financial distress. Long and Malitz (1985), among others, propose this explanation for a negative link between leverage and asset risk. To provide unique evidence for the beta anomaly channel, we put more weight on the following hypothesis.

Hypothesis 2. Leverage is negatively related to both upside and downside beta.

Under a standard tradeoff where financial distress reduces the attractiveness of debt, all else equal, one expects a negative relationship between leverage and proxies for distress risk, such as, potentially, asset beta. Observation 4 helps us rule this alternative explanation out.

Observation 4 is based on equation (17) which says that there is a linear beta anomaly: The slope γ is constant across the full distribution of β, as it is in the model of Brennan (1993) and as emphasized by Baker et al. (2011). Moreover, if anything, other theories suggests that the beta anomaly might be stronger in the range of positive β^+. Hong and Sraer (2016) focus on the interaction between overconfidence and high beta, or more speculative, stocks. These stocks have greater disagreement and, under short sales constraints, are more overvalued. A richer model might feature greater overconfidence in periods of rising market returns and in principle deliver a stronger risk anomaly in upside betas.

This prediction is separate from the traditional tradeoff theory and the ideas in Almeida and Philippon (2007) or Shleifer and Vishny (1992). The costs of financial distress depend not only on the unconditional probability of default and value lost in default, but also when distress occurs and value is lost. If asset beta, holding all else constant, including total risk, dictates the market state when distress is likely to occur, then the present value of the costs of financial distress are higher for assets with higher systematic risk. Almeida and Philippon argue that risk-adjustment increases the cost of financial distress, while Shleifer and Vishny

offer the tangible example of refinancing risk and fire sales. If refinancing risks and fire sale discounts are higher during market downturns, this would increase the value lost in distress and lower optimal leverage for firms with higher levels of systematic risk, although it is still hard to justify zero debt in the presence of large tax benefits.

Put simply, in the traditional tradeoff, "downside risk" is the emphasis. Risk matters because of bankruptcy costs (if anything, upside risk actually tends to increase optimal leverage by increasing expected tax benefits), and this is a plausible effect. For our purpose, the point is that if a beta risk version of the traditional tradeoff drives an empirical link between high asset beta and leverage, it should be concentrated in downside risk. Our concern is to distinguish the beta anomaly tradeoff, in which upside and downside beta are equally relevant and *both* (most tellingly, upside beta) should be negatively related to leverage.

Hypothesis 3. Leverage is negatively related to equity beta.

This is based on Observation 5, which notes that a negative empirical relationship between equity beta and leverage is sufficient to prove a negative empirical relationship between asset beta and leverage, our main point. Again, the utility of this prediction is that it allows us to avoid conditioning any negative leverage-asset beta empirical relationship on the assumption of zero debt beta which, as an empirical matter, is a common assumption and almost unavoidable. In our tests, we consider both the financial distress risk alternative explanation and the zero debt beta assumption at once by relating leverage to upside and downside equity beta.

B. Data

Our main variables are introduced in Table 1. Our basic sample is the portion of the merged CRSP-Compustat sample for which marginal tax rates are available from John Graham. The data begin in 1980, when marginal tax rates are first available, and end in 2014. They contain 1,038,097 firm-months and span all 50 Fama–French (1997) industries. Unlike much capital structure research, we include financial firms because they are not special under the beta anomaly theory, but their exclusion does not affect the relevant results. In an average cross section there are 2,181 profitable and 291 unprofitable firms.

Variable definitions are in the Appendix. Gross book leverage is long-term debt and notes payable divided by the sum of long-term debt and notes payable plus book equity. Net book leverage nets out cash and equivalents from the numerator and denominator. Gross and net market leverage replace book equity with the market value of common equity from CRSP.

The regressions control for traditional explanatory variables in Bradley, Jarrell, and Kim (1984), Rajan and Zingales (1995), Frank and Goyal (2009), Campello and Giambona (2013), and others. The fixed assets ratio, a proxy for financial distress costs, is net property, plant, and equipment divided by total assets. Profitability, which would be positively correlated with leverage under the standard tradeoff theory but inversely correlated under the Myers and Majluf (1984) pecking order, is EBIT divided by total assets. Market-to-book assets is known to be negatively related to leverage, consistent with the need for firms with strong

TABLE 1
Summary Statistics: CRSP and Compustat Data

Table 1 reports leverage ratios, asset beta and risk, and capital structure determinants over the period from 1980 to 2014. We divide firms into profitable and unprofitable. A firm is defined as profitable if it has earnings before interest and taxes (Compustat = EBITDA) greater than 0. Variable definitions are provided in the Appendix. There are 974,470 observations in 50 industries across 420 months.

	Profitable Firms			Unprofitable Firms		
	Avg. N	Mean	Std. Dev.	Avg. N	Mean	Std. Dev.
BOOK_LEV_GROSS%	2,066	32.7	25.6	255	30.1	33.1
MARKET_LEV_GROSS%	2,066	26.8	24.5	255	21.5	25.8
BOOK_LEV_NET%	2,066	21.3	33.4	255	12.9	41.7
MARKET_LEV_ NET%	2,066	18.4	29.2	255	8.9	31.6
ASSET_BETA	2,066	0.71	0.54	255	0.89	0.81
UPSIDE_ASSET_BETA	2,066	0.58	0.56	255	0.58	0.77
DOWNSIDE_ASSET_BETA	2,066	0.83	0.61	255	1.15	0.99
IND_UPSIDE_ASSET_BETA	2,066	0.79	0.30	255	0.88	0.33
IND_DOWNSIDE_ASSET_BETA	2,066	0.84	0.30	255	0.93	0.32
UPSIDE_EQUITY_BETA	2,066	0.66	0.52	255	0.53	0.60
DOWNSIDE_EQUITY_BETA	2,066	0.96	0.49	255	1.06	0.62
PRE_INT_MGL_TAX_RATE%	2,066	33.4	10.3	255	14.0	14.0
FIXED_ASSETS_RATIO (%)	2,066	32.4	23.6	255	24.3	22.4
PROFITABILITY (%)	2,066	9.7	7.6	255	−21.5	20.4
MARKET_TO_BOOK_ASSETS	2,066	1.9	1.9	255	3.1	3.9
ln(ASSETS)	2,066	5.7	2.2	255	3.4	1.8
ASSET_GROWTH (%)	2,066	13.8	29.6	255	3.3	46.2

growth opportunities to avoid having to pass them up (Myers (1977)) as well a more passive lack of adjustment of leverage to prior stock returns (Welch (2004)). This variable is gross debt and market equity divided by the sum of gross debt and book equity. Asset growth could be a proxy for growth opportunities, or capture size, or the profitability that helps to make debt-financed acquisitions. Size, the natural log of book assets, may also proxy for multiple influences. Fama and French (1992) use it to represent the greater cash flow volatility of smaller firms and their higher expected costs of financial distress. It will also reflect their lesser access to debt markets. Finally, Graham's pre-interest marginal tax rates account for many features of the tax code. As shown by Graham and Mills (2008), these approximate the tax rates simulated with federal tax return data.

The leverage determinants that interest us most are constructed from stock returns. Asset beta is unlevered equity beta, assuming debt is riskless. While betas on corporate debt are very low, they are difficult to observe. To avoid the results depending on this assumption we have Hypothesis 3. Total equity risk is the standard deviation of excess stock returns. Asset risk is the unlevered version. Industry asset beta and risk are market equity-weighted averages.

C.   Summary Statistics

Tables 2 and 3 report summary statistics and correlations. Profitable firms are larger and have higher tax rates. Asset beta is somewhat higher for unprofitable firms, as is total risk, which we will use as a control variable for financial distress costs. With respect to asset risk, a firm must be promising and at least on a path to profitability to enter the CRSP-Compustat sample for the 24 months that we require to compute beta. Becoming unprofitable may be associated with

TABLE 2
Correlations: CRSP and Compustat Data

Table 2 reports leverage ratios, asset beta and risk, and capital structure determinants over the period 1980–2014. Variable definitions are provided in the Appendix. There are 974,470 observations in 50 industries across 420 months.

Panel A. Leverage Ratio

	Book Leverage		Market Leverage	
	Gross	Net	Gross	Net
BOOK_LEV_GROSS%	1.00			
MARKET_LEV_GROSS (%)	0.78	1.00		
BOOK_LEV_NET (%)	0.93	0.77	1.00	
MARKET_LEV_NET (%)	0.78	0.93	0.86	1.00

Panel B. Leverage Determinants

	Own Asset Risk				Industry Asset Risk			Own Equity Risk		
	Beta	Up	Down	Std. Dev.	Up	Down	Std. Dev.	Up	Down	Std. Dev.
ASSET_BETA	1.00									
UPSIDE_ASSET_BETA	0.87	1.00								
DOWNSIDE_ASSET_BETA	0.93	0.65	1.00							
ASSET_RISK (%)	0.58	0.34	0.66	1.00						
IND_UPSIDE_ASSET_BETA	0.37	0.33	0.35	0.29	1.00					
IND_DOWNSIDEASSET_BETA	0.34	0.27	0.35	0.30	0.91	1.00				
IND_ASSET_RISK (%)	0.29	0.22	0.29	0.34	0.75	0.83	1.00			
UPSIDE_EQUITY_BETA	0.62	0.82	0.37	−0.01	0.23	0.17	0.10	1.00		
DOWNSIDE_EQUITY_BETA	0.63	0.45	0.69	0.18	0.26	0.28	0.18	0.53	1.00	
RISK (%)	0.17	0.02	0.25	0.61	0.23	0.27	0.33	−0.03	0.28	1.00
PRE_INT_MGL_TAX_RATE%	−0.07	0.01	−0.12	−0.37	−0.12	−0.12	−0.16	0.09	−0.03	−0.47
FIXED_ASSETS_RATIO (%)	−0.22	−0.16	−0.23	−0.25	−0.28	−0.25	−0.24	−0.06	−0.10	−0.15
PROFITABILITY (%)	−0.02	0.06	−0.07	−0.30	−0.06	−0.06	−0.09	0.09	−0.05	−0.42
MARKET_TO_BOOK_ASSETS	0.19	0.15	0.19	0.18	0.18	0.18	0.17	0.10	0.14	0.18
ln(ASSETS)	0.12	0.28	−0.02	−0.41	−0.11	−0.15	−0.12	0.47	0.16	−0.51
ASSET_GROWTH (%)	0.07	0.05	0.08	0.03	0.03	0.05	0.03	0.05	0.10	0.04

unexpectedly negative returns. Also, firms in variable industries are more likely to find themselves unprofitable in a given period. The latter logic also applies to beta, on the downside.

Other notable correlations in Table 2 are as follows: Gross and net leverage measures are loosely correlated enough to consider both as a robustness exercise. We follow tradition and consider both book and market leverage measures. Asset beta, for the own firm or the industry, is negatively correlated with tax rates and fixed assets and positively correlated with market-to-book and size. These correlations are generally small relative to the correlations among the various risk measures.

D.   Extreme Leverage

Although firms at the leverage extremes are not uncommon, they are particularly interesting to consider in light of the beta anomaly because they are where the standard tradeoff theory is least compelling. In particular, a beta anomaly tradeoff could help to explain some of the low leverage puzzle of Graham (2000). As an example, Linear Technology Corporation (NASDAQ: LLTC) produces semiconductors with a market capitalization of $7.7 billion as of Dec. 2012. Despite profitable operations, a pre-interest marginal tax rate of 35% by the methodology in Graham and Mills (2008), and a cash balance of $1 billion, Linear maintains negative net debt. One potential explanation for this may be its high asset beta.

TABLE 3

Summary Statistics for Profitable Firms: CRSP and Compustat Data

Table 3 presents leverage ratios, asset beta and risk, and capital structure determinants over the period 1980–2014. We divide the sample of profitable CRSP-Compustat firms into 6 groups, according to gross book leverage (in Panels A and B) and according to pre-interest marginal tax rate (across 3 pairs of columns). A firm is defined as profitable if it has earnings before interest and taxes (Compustat = EBITDA) greater than 0. The marginal tax rate is from John Graham, computed using the methodology of Graham and Mills (2008). Variable definitions are provided in the Appendix.

				Tax Rates		
	Avg. N	MTR < 5%	Avg. N	Middle	Avg. N	MTR > 30%
Panel A. Low Leverage, <5% Gross Book Leverage						
BOOK_LEV_GROSS%	17	0.7	55	0.9	283	1.0
MARKET_LEV_GROSS%	17	0.7	55	0.8	283	0.7
BOOK_LEV_NET%	17	−20.4	55	−19.9	283	−20.0
MARKET_LEV_ NET%	17	−16.1	55	−17.0	283	−13.9
ASSET_BETA	17	1.22	55	1.14	283	1.05
UPSIDE_ASSET_BETA	17	0.89	55	0.83	283	0.85
DOWNSIDE_ASSET_BETA	17	1.50	55	1.40	283	1.21
IND_UPSIDE_ASSET_BETA	17	0.94	55	0.94	283	0.91
IND_DOWNSIDE_ASSET_BETA	17	0.99	55	0.98	283	0.95
UPSIDE_EQUITY_BETA	17	0.62	55	0.58	283	0.70
DOWNSIDE_EQUITY_BETA	17	1.04	55	0.97	283	0.96
PRE_INT_MGL_TAX_RATE%	17	1.9	55	17.8	283	36.6
FIXED_ASSETS_RATIO (%)	17	22.0	55	21.6	283	22.0
PROFITABILITY (%)	17	4.8	55	6.4	283	14.7
MARKET_TO_BOOK_ASSETS	17	2.82	55	2.13	283	2.98
ln(ASSETS)	17	4.06	55	3.97	283	4.86
ASSET_GROWTH (%)	17	17.6	55	11.3	283	15.1
Panel B. High Leverage, >50% Gross Book Leverage						
BOOK_LEV_GROSS%	19	75.9	78	74.3	371	67.9
MARKET_LEV_GROSS%	19	57.7	78	58.2	371	54.8
BOOK_LEV_NET%	19	71.1	78	69.8	371	63.1
MARKET_LEV_ NET%	19	54.1	78	54.3	371	50.9
ASSET_BETA	19	0.37	78	0.37	371	0.41
UPSIDE_ASSET_BETA	19	0.25	78	0.26	371	0.33
DOWNSIDE_ASSET_BETA	19	0.48	78	0.45	371	0.48
IND_UPSIDE_ASSET_BETA	19	0.75	78	0.73	371	0.68
IND_DOWNSIDE_ASSET_BETA	19	0.81	78	0.79	371	0.73
UPSIDE_EQUITY_BETA	19	0.54	78	0.55	371	0.65
DOWNSIDE_EQUITY_BETA	19	1.03	78	0.97	371	0.97
PRE_INT_MGL_TAX_RATE%	19	2.2	78	17.9	371	37.1
FIXED_ASSETS_RATIO (%)	19	37.3	78	37.2	371	35.3
PROFITABILITY (%)	19	2.1	78	4.2	371	7.7
MARKET_TO_BOOK_ASSETS	19	1.96	78	1.74	371	1.58
ln(ASSETS)	19	5.34	78	5.29	371	6.37
ASSET_GROWTH (%)	19	17.6	78	7.5	371	17.0

While rarer than inexplicably low-leverage firms, a number of profitable firms maintain high leverage despite little tax benefit. Under a standard tradeoff theory, this amounts to needlessly tempting a fate of financial distress. An example is Textainer (NYSE: TGH), a firm that leases and trades marine cargo containers. As of the end of 2012, its market capitalization was approximately $1.7 billion. It has tangible assets of $3.4 billion and a cash balance of $175 million. Despite a marginal tax rate close to 0%, as a result of front-loaded depreciation, modest growth, and an offshore tax status, it maintains $2.7 billion in debt. A potential explanation for this inconsistency with the standard tradeoff theory is the firm's low asset beta. Under a beta anomaly, equity is undervalued at low leverage, and its value rises steadily as leverage increases to its correct valuation, and potentially beyond.

The beta anomaly tradeoff may also be pertinent to a set of uniquely highly leveraged firms (banks) which are often excluded from capital structure analyses.

Figure 1 suggests that a beta anomaly in equities means that regulating low asset beta firms, in the sense of requiring them to delever significantly, can impose large increases in the cost of capital and losses in shareholder value. Baker and Wurgler (2015) find that banks' asset betas are on the order of 0.10, and that the beta anomaly within banks is at least as large as for all firms. While there are numerous other forces at play in regulatory debates, the loss of the beta anomaly's benefits gives a coherent foundation for bankers' common argument that reducing leverage would increase their cost of capital.

Table 3 looks more closely within profitable firms, where we have 867,524 observations and the shortcomings of the standard tradeoff theory appear most clearly. The panels separate profitable firms into low leverage (gross book leverage <5%) and high leverage (gross book leverage >50%) groups. What counts as high leverage is subjective. We obviously cannot expect a mode at 100% that resembles the mode at 0%, so we choose an arbitrary cutoff of 50%. The columns then add another sort into low (MTR<5%), medium, and high (MTR>30%) marginal tax rate groups.

The low leverage puzzle is represented in the large number of firm-months that have chosen very low leverage despite positive profitability and high marginal tax rates. In the average cross section, not far from half of firms in the highest tax category, 43%, have chosen low leverage over high leverage (43%=283/(283+371)). Firms like Linear Technology are here. The high leverage puzzle, if we can call it that for sake of illustration, is the narrower but still noticeable fact that nearly half of the firms in the low tax category, 47%, have chosen high leverage (47%=19/(17+19)). Firms like Textainer are in this bin.

Of course, this is certainly not the only potential driver of extreme leverage. For example, Denis and McKeon (2012) explain it as an outcome of the evolution of operating needs and the desire to maintain financial flexibility (see also Hackbarth and Mauer (2012)). Hackbarth (2009) suggests a role for managerial optimism. Regression results follow below, but some initial support for the beta anomaly tradeoff as an incremental influence comes from the much stronger differences in asset risk across the leverage levels. Within the middle tax rate group, for example, asset betas decline sharply with leverage. Firms with very low leverage have a median asset beta of 1.14, a median upside asset beta of 0.83, and a median downside asset beta of 1.40. For high leverage firms this falls to 0.37, 0.26, and 0.45, respectively.

E.   Regressions

Turning from extreme leverage observations to the broader cross section, the first column in Panel A of Table 4 reports a baseline gross book leverage regression using typical covariates. We report marginal effects of Tobit regressions that cluster on both firm and month. The first several variables' signs and effects are consistent with prior research, as is the poor overall R^2. The marginal tax rate has a positive coefficient, fixed assets a fairly strong positive coefficient, profitability a negative coefficient, market-to-book a negative coefficient, and size a positive coefficient. Rajan and Zingales (1995) focus on the latter 4 variables and obtain the same results. Asset growth has a positive coefficient, more consistent with the interpretation that asset growth is a consequence of the ability and desire to finance

TABLE 4

Capital Structure and Asset Risk (1980–2014)

Table 4 reports Tobit regressions of gross book leverage on capital structure determinants. Gross leverage ratio is defined as long-term debt (DLTT) plus notes payable (NP) divided by long-term debt plus notes payable plus book equity. Book equity is computed in the same way as in Kenneth French's data library. Robust t-statistics, with standard errors clustered by month and by industry, are in brackets. Regressions labeled "Own Risk Measures" use firm measures of asset beta and asset risk. Regressions labeled "Industry Risk Measures" use matched Fama–French industry measures of asset beta and asset risk. Other variable definitions are in the Appendix.

	Base		Own Risk Measures				Industry Risk Measures			
	Coef.	t-Stat.	Coef.	t-Stat.	Coef.	t-Stat.	Coef.	t-Stat.	Coef.	t-Stat.
Panel A. Asset Beta										
ASSET_BETA			−11.6	[−13.0]	−12.6	[−10.6]	−15.8	[−4.5]	−9.4	[−2.5]
ASSET_RISK (%)			−2.74	[−16.7]	−2.65	[−13.3]	−2.39	[−2.6]	−2.44	[−2.4]
PRE_INT_MGL_TAX_RATE%	0.11	[2.4]			−0.11	[−3.0]			0.06	[1.4]
FIXED_ASSETS_RATIO (%)	0.19	[4.1]			0.05	[1.6]			0.14	[3.5]
PROFITABILITY (%)	−0.40	[−8.2]			−0.46	[−13.5]			−0.39	[−8.2]
MARKET_TO_BOOK_ASSETS	−1.4	[−5.7]			−0.3	[−1.3]			−1.1	[−5.0]
ln(ASSETS)	2.8	[8.8]			2.0	[6.4]			2.6	[10.3]
ASSET_GROWTH (%)	0.04	[4.4]			0.06	[9.4]			0.04	[4.7]
2-way clustering		Yes		Yes		Yes		Yes		Yes
Industries		50		50		50		50		50
Months		420		420		420		420		420
N (000)		974		974		974		974		974
Panel B. Upside and Downside Asset Beta										
UPSIDE_ASSET_BETA			−6.4	[−10.7]	−8.1	[−11.7]	−14.7	[−5.2]	−12.7	[−3.9]
DOWNSIDE_ASSET_BETA			−8.2	[−12.4]	−7.5	[−12.1]	−0.3	[−0.1]	5.1	[1.5]
UPSIDE_ASSET_RISK (%)			−1.71	[−4.7]	−0.69	[−1.4]	−3.55	[−1.0]	4.84	[1.1]
DOWNSIDE_ASSET_RISK (%)			−3.15	[−6.3]	−4.06	[−8.2]	−3.07	[−0.9]	−12.22	[−3.0]
Controls	Yes		Yes		Yes		Yes		Yes	
2-way clustering	Yes		Yes		Yes		Yes		Yes	
Industries	50		50		50		50		50	
Months	420		420		420		420		420	
N (000)	974		974		974		974		974	

with debt, determined by other underlying sources, than an additional proxy for growth opportunities.

1. Leverage and Asset Beta

We now add risk measures to test Hypothesis 1. Our special focus is on asset beta, which is what the beta anomaly tradeoff suggests, but we also control for overall risk. In principle, any effect of total asset risk could reflect the beta anomaly tradeoff; some explanations of the beta anomaly are specific to beta, others are not. Total asset risk is a plausible proxy for the expected costs of financial distress, especially compared to asset beta.

The middle columns of Panel A show that asset beta is a strong determinant of leverage, consistent with Hypothesis 1. This is true controlling for overall asset risk (and in unreported univariate regressions). Adding the control variables does not significantly affect the coefficient or t-statistic on asset beta. With all controls, a 1-unit increase in asset beta reduces leverage by 12.6%, a large effect by the standards of the control variables.

The last columns of Table 4 show that the economic effects remain if we use industry risk, which is an empirical solution to the issue of any mechanical negative link between leverage and asset beta created by using leverage itself to unlever equity beta. It appears that any measurement error introduced by this switch does

not appear to greatly affect the coefficients on beta risk. These regressions provide further support for Hypothesis 1.

2.   Upside and Downside Asset Beta

We turn next to Hypothesis 2. It is clear that high asset beta is associated with lower leverage. This is consistent with the beta anomaly tradeoff whereby the cost of equity for high beta assets is lower and so less debt is optimal, but also consistent with versions of the standard tradeoff theory to the extent that the relationship is driven by downside risk. To examine whether downside risk alone is driving the relationship, we estimate equity beta separately over months when the market risk premium was positive and when it was negative. Unlevering these and averaging by industry gives us an upside asset beta and downside asset beta measure.

In Panel B of Table 4 we find that the upside and downside components of asset beta have an equally strong relationship to gross book leverage when using the firm's own risk measures. Furthermore, the relationship actually favors the upside asset beta under the preferred industry-based measures. Downside asset beta has no statistical association with leverage and in one case a point estimate of the wrong sign to support a version of the traditional tradeoff.

Reassuringly for the traditional tradeoff, and consistent with our use of asset risk as a control variable for financial distress, the downside overall asset risk association with leverage is generally stronger than the upside asset risk association. This of course does not alter the conclusions relevant to Hypothesis 2, a prediction involving asset beta risk. To repeat, our goal is not to cast doubt on the relevance of the traditional tradeoff theory, but to show that there is a connection between leverage and asset beta that is less tortured to explain by the beta anomaly.

Support for Hypotheses 1 and 2 also appears under other leverage measures, including gross market leverage and net book and market leverage, still using industry risk measures. Panel A of Table 5 shows results consistent with only a beta anomaly tradeoff, in the form of an upside beta effect, remain strong, as well as results consistent with both a beta anomaly tradeoff and a traditional tradeoff, remain in the form of downside risk effects.

3.   Upside and Downside Equity Beta

We now test Hypothesis 3, which addresses the required practical assumption of a zero debt beta. While debt betas in practice are very low, Hypothesis 3, which is based on the theoretical observation that a negative empirical relationship between leverage and equity beta is sufficient to prove a negative empirical relationship between leverage and asset beta, gives us a more grounded way to avoid conditioning our asset beta results on this assumption.

The results in Panel B of Table 5 confirm a negative relationship between leverage and equity beta. The result that upside beta risk is much stronger than downside beta risk remains. It is worth noting that any mechanical link between leverage and equity beta would bias results in a positive direction (i.e., away from our hypotheses of interest), so it is particularly comforting that the results remain consistent with the beta anomaly tradeoff.

TABLE 5
Alternate Leverage Ratios and Equity Beta (1980–2014)

Table 5 reports Tobit regressions of leverage on capital structure determinants. We repeat the regression of the third column of Panel B in Table 4, using 4 different measures of leverage. Net leverage ratios deduct cash and equivalents from debt. Market leverage ratios replace book equity with market capitalization, equal to price times shares outstanding from CRSP. Panel B replaces asset measures of beta and risk with equity measures of beta and risk. Robust t-statistics, with standard errors clustered by month and by industry, are in brackets.

| | Gross Leverage (%) | | | | Net Leverage (%) | | | |
| | Book | | Market | | Book | | Market | |
	Coef.	t-Stat.	Coef.	t-Stat.	Coef.	t-Stat.	Coef.	t-Stat.
Panel A. Upside and Downside Asset Beta								
UPSIDE_ASSET_BETA	−8.1	[−11.7]	−8.4	[−12.0]	−13.1	[−11.3]	−12.6	[−13.7]
DOWNSIDE_ASSET_BETA	−7.5	[−12.1]	−8.0	[−13.3]	−16.5	[−12.9]	−16.3	[−15.5]
UPSIDE_ASSET_RISK (%)	−0.69	[−1.4]	−1.05	[−2.7]	−3.99	[−7.8]	−4.50	[−10.4]
DOWNSIDE_ASSET_RISK (%)	−4.06	[−8.2]	−3.36	[−7.9]	−8.70	[−12.9]	−7.86	[−15.9]
Controls		Yes		Yes		Yes		Yes
2-way clustering		Yes		Yes		Yes		Yes
Industries		50		50		50		50
Months		420		420		420		420
N (000)		974		974		974		974
Panel B. Upside and Downside Equity Beta								
UPSIDE_EQUITY_BETA	−7.3	[−7.4]	−8.1	[−9.6]	−10.3	[−6.4]	−10.6	[−8.1]
DOWNSIDE_EQUITY_BETA	1.5	[1.5]	0.5	[0.7]	0.5	[0.4]	−0.1	[−0.1]
UPSIDE_EQUITY_RISK (%)	5.80	[9.0]	4.84	[8.5]	6.61	[7.9]	5.51	[8.1]
DOWNSIDE_EQUITY_RISK (%)	−0.99	[−1.5]	−0.45	[−0.8]	−1.03	[−1.3]	−0.48	[−0.8]
Controls		Yes		Yes		Yes		Yes
2-way clustering		Yes		Yes		Yes		Yes
Industries		50		50		50		50
Months		420		420		420		420
N (000)		974		974		974		974

IV.   Other Robustness Tests

We find qualitatively identical results using a variety of other asset beta estimation approaches. Acharya, Almeida, and Campello (2013) use a structural model to estimate beta. They kindly shared their data, and we also replicated their methodology in our full data set and jointly estimated asset volatility with asset beta. Choi (2013) uses an empirical measure of asset betas by combining the returns of fixed income instruments with equity weighted by their market values. These approaches all lead to qualitatively identical results to our simple and transparent estimate of asset betas. The detailed results are available on request.

Finally, in additional unreported robustness tests suggested by referees, we found that switching to quarterly data (instead of clustering monthly), or expanding the sample by dropping the requirement to include Graham's marginal tax rates, had little effect. Dropping total risk or equity risk in Tables 4 and 5 also had no qualitative impact.

V.   Conclusion

Many studies have shown that high-risk equities do not earn commensurately high returns. This paper derives a novel explanation for leverage that is consistent with this observation. We show that for firms with relatively risky assets, the cost of capital is minimized at a low level of leverage. For firms with very low risk assets, low leverage entails a substantial cost in the form of issuing undervalued

equity, and hence the cost of capital is minimized at much higher levels of leverage. In the data, leverage is indeed inversely related to systematic risk, supporting the main prediction of the beta anomaly tradeoff.

Importantly, we derive and test a prediction of the beta anomaly tradeoff that allows us to separate it from a standard tradeoff explanation in which asset beta reflects financial distress costs. The beta anomaly tradeoff predicts that leverage is inversely related to upside risk, not just downside risk. This is also confirmed. In some specifications, the relationship between leverage and upside risk is stronger than the relationship with downside risk.

More broadly, we suggest that the beta anomaly tradeoff may help to explain the low and high leverage puzzles in which leverage choices cannot be easily explained by tax and financial distress considerations alone. Better understanding the empirical limits of a beta anomaly tradeoff, and how it complements other realistic explanations for leverage that receive support in the literature, is an area for future research. Moreover, while the beta anomaly is perhaps the obvious starting point, fruitful research might also come from investigating the interplay of capital structure and numerous other stock market anomalies.

Appendix. Variable Definitions

All variables are winsorized at 1% and 99% as measured across the whole sample.

ASSET_BETA: EQUITY_BETA times 1 minus net market leverage.

ASSET_GROWTH: The annual change in total assets (ASSETS) divided by total assets 1 year ago, in percentage terms.

ASSET_RISK (%): EQUITY_RISK times 1 minus net market leverage.

BOOK_EQUITY: Shareholder's equity minus preferred stock plus deferred taxes. Shareholder's equity (SEQ) or the sum of common equity (CEQ) plus preferred stock (PSTK) if shareholder's equity is missing or total assets (ASSETS) minus total liabilities (LT) if common equity is missing. Preferred stock is equal to the redemption value of preferred stock (PSTKRV) or the liquidating value of preferred stock (PSTKL) or total preferred stock (PSTK) in that order and is set to 0 if still missing. Deferred taxes are equal to deferred tax and investment tax credit (TXDITC) or balance sheet deferred tax (TXDB) in that order and 0 if missing.

BOOK_LEV_GROSS%: The sum of total long-term debt (COMPUSTAT = DLTT) and notes payable (NP) divided by the sum of total long-term debt and notes payable and book equity, in percentage terms.

BOOK_LEV_NET%: The sum of total long-term debt (COMPUSTAT = DLTT) and notes payable (NP) less cash and equivalents (CHE) divided by the sum of total long-term debt and notes payable and book equity less cash and equivalents, in percentage terms.

DOWNSIDE_ASSET_BETA: DOWNSIDE_EQUITY_BETA times 1 minus net market leverage.

DOWNSIDE_ASSET_RISK (%): DOWNSIDE_EQUITY_RISK times 1 minus net market leverage.

DOWNSIDE_EQUITY_BETA: Market beta computed from CRSP returns (RET) net of Treasury bill returns (YLDMAT) from CRSP regressed on the value-weighted market return (VWRET), also net of the Treasury bill return, times an indicator variable

when the realized market return is positive and the value-weighted market return times an indicator variable when the realized market return is negative, using 3-day overlapping return windows. We require at least 750 overlapping windows of returns and use at most 5 years of returns. The downside beta is the coefficient on negative market returns.

DOWNSIDE_EQUITY_RISK (%): Standard deviation of CRSP returns (RET) net of Treasury bill returns (YLDMAT), in percentage terms, conditional on the CRSP return (RET) net of the Treasury bill return being negative.

EQUITY_BETA: Market beta computed from CRSP returns (RET) net of Treasury bill returns (YLDMAT) from CRSP regressed on the value-weighted market return (VWRET), also net of the Treasury bill return, using 3-day overlapping return windows. We require at least 750 overlapping windows of returns and use at most 5 years of returns.

EQUITY_RISK (%): Standard deviation of CRSP returns (RET) net of Treasury bill returns (YLDMAT), in percentage terms.

FIXED_ASSETS_RATIO (%): Plant, property, and equipment, net (PPENT) divided by total assets (ASSETS), in percentage terms.

IND_ASSET_BETA: Market equity weighted average asset beta, computed for each Fama–French industry classification. Market equity is equal to price (PRC) times shares outstanding (CRSP) from CRSP. The 49 industry classifications are defined in Ken French's data library, with unclassified firms comprising a 50th group.

IND_ASSET_RISK (%): Market equity weighted average asset risk, computed for each Fama–French industry classification. Market equity is equal to price (PRC) times shares outstanding (CRSP) from CRSP. The 49 industry classifications are defined in Kenneth French's data library, with unclassified firms comprising a 50th group.

ln(ASSETS): The natural log of total assets (ASSETS).

MARKET_TO_BOOK_ASSETS: Sum of total long-term debt (COMPUSTAT = DLTT) and notes payable (NP) and market equity divided by the sum of total long-term debt and notes payable and book equity. Market equity is equal to price (PRC) times shares outstanding (CRSP) from CRSP.

MARKET_LEV_GROSS%: The sum of total long-term debt (COMPUSTAT = DLTT) and notes payable (NP) divided by the sum of total long-term debt and notes payable and market equity. Market equity is equal to price (PRC) times shares outstanding (CRSP) from CRSP, in percentage terms.

MARKET_LEV_NET%: The sum of total long-term debt (COMPUSTAT = DLTT) and notes payable (NP) less cash and equivalents (CHE) divided by the sum of total long-term debt and notes payable and market equity less cash and equivalents. Market equity is equal to price (PRC) times shares outstanding (CRSP) from CRSP, in percentage terms.

PRE-INT_MGL_TAX_RATE%: John Graham provided estimates of the pre-interest marginal tax rate, computed using the methodology of Graham and Mills (2008), in percentage terms.

PROFITABILITY%: Earnings before interest and taxes (EBIT) divided by total assets (ASSETS), in percentage terms.

UPSIDE_ASSET_BETA: UPSIDE_EQUITY_BETA times 1 minus net market leverage.

UPSIDE_ASSET_RISK%: UPSIDE_EQUITY_RISK times 1 minus net market leverage.

UPSIDE_EQUITY_BETA: See DOWNSIDE_BETA. The upside beta is the coefficient on positive market returns.

UPSIDE_EQUITY_RISK%: Standard deviation of CRSP returns (RET) net of Treasury bill returns (YLDMAT), in percentage terms, conditional on the CRSP return (RET) net of the Treasury bill return being positive.

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