Estimating Industry Multiples

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Abstract

We analyze industry multiples for the S&P 500 in 1995. We use Gibbs sampling to estimate simultaneously the error specification and small sample minimum variance multiples for 22 industries. In addition, we consider the performance of four common multiples: the simple mean, the harmonic mean, the value-weighted mean, and the median. The harmonic mean is a close approximation to the Gibbs minimum variance estimates. Finally, we show that EBITDA is a better single basis of substitutability than EBIT or revenue in the industries that we examine.

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1. Introduction

Valuing firms as a multiple of a financial or operating performance measure is a simple, popular, and theoretically sound approach to valuation. It can be used by itself, or as a supplement to a discounted cash flow approach. It applies the only the most basic concept in economics: perfect substitutes should sell for the same price. The basis of substitutability is generally a measure of financial or operating performance and the multiple is the market price of a single unit. If, for example, the basis of substitutability is revenue, then the revenue multiple is the market price of a dollar of revenue. Multiplying the amount of revenue by the revenue multiple provides an estimate of value. The basis of substitutability can be a financial measure from either the income statement or the balance sheet or an operating measure, such as potential customers, subscribers, or established reserves.

The method of multiples has advantages over the discounted cash flow method. Implicit in the multiple is a forecast of future cash flows and an estimate of the appropriate discount rate. The method of multiples uses current market measures of the required return and industry growth rates. It avoids the problems in applying the discounted cash flow techniques of selecting a theoretical model of the appropriate discount rate and estimating it using historical data. It also avoids the difficulty of independently determining future cash flows. If a truly comparable publicly traded firm or transaction were available, if the basis of substitutability could be determined, and if the multiple could be estimated reliably, then the method of multiples would be clearly superior to discounted cash flow analysis.

The drawback to the method of multiples is in its three implementation challenges. The first is determining the basis of substitutability. Typically, the basis of substitutability is chosen qualitatively as some measure of financial performance, such as revenue, earnings before interest, taxes, and depreciation (EBITDA), or cash flow, or a measure of operating performance, such as established reserves or subscribers. The second implementation challenge is measuring the multiple. Practitioners generally use the simple mean or median of the multiples implicit in the market pricing of a set of publicly traded comparable firms or comparable publicly disclosed transactions. The third implementation challenge is choosing a set of comparable companies or transactions.

This paper focuses on the first two implementation challenges of the method of multiples. We abstract from the difficulty of choosing a set of appropriate comparable firms or transactions and focus on the econometric problem of inferring the industry multiple and basis of substitutability. There are two related econometric problems. The first is that estimating multiples from comparable companies or transactions invariably involves a small number of observations. In our study, for example, we have only 10 firms on average across the 22 S&P industries we examine. The small number of observations means that standard approaches, like ordinary least squares regression analysis, do not lead to minimum variance estimators. The second problem is that the statistical errors from the multiple valuation model are unlikely to be homoskedastic. Firms with different values may have different error variances, and the minimum variance estimator depends on the form of this variance.

Section 2 presents our econometric approach. Our aim is to determine the minimum variance estimate for the industry multiple M. The method of multiples, as typically used in practice, presumes a linear relationship within a given industry between value and the basis of substitutability:

$$V_j = MX_j + \varepsilon_j \quad \text{for } j = 1, \ldots, N \tag{1}$$

where V and X denote the value and a measure of financial or operating performance for firm j, M is a multiple that is constant across N firms of a particular industry, and ε is an error which reflects the variation in multiples across firms within an industry.

The minimum variance industry multiple depends on the specification of the errors. We assume that the error ε is proportional to a power of the value V:

$$\varepsilon_j \sim V_j^{\lambda} \tag{2}$$

We show that this error specification is both conceptually and empirically reasonable. Conceptually, the errors from a growing perpetuity of cash flows valuation model will depend on the value of the firm. Empirically, we show that, when (1) is transformed to account for the dependence of the errors on firm value, the resulting errors appear to be normally distributed.

In section 3, we use Gibbs sampling to simultaneously estimate the multiple and the error structure. We use data for 22 S&P 500 industry groups in 1995. We restrict the model so that the errors are related to value in the same way across all of the firms within all of the industries. This method provides the minimum variance estimate of the multiple in (1). The Gibbs results also show that the standard deviation of the errors is linearly related to the value of the firm.

Although the Gibbs estimation provides a multiple for each industry, we view it as an impractical method of estimating multiples for practitioners. The Gibbs approach requires that the error structure and the multiples be estimated simultaneously and thereby requires data across industries. The typical practitioner problem, however, involves estimating a multiple for a single industry or group of transactions. In section 4, we therefore also consider four common estimators of multiples: the simple mean, the harmonic mean, the value-weighted mean, and the median. The harmonic mean is calculated by averaging the inverse of the multiples $\frac{X}{V}$, and taking the inverse of that average. The value-weighted mean is the total industry market value $\sum V$ divided by the industry total for the basis of substitutability $\sum X$.

Figure 1 plots the simple mean, the harmonic mean, the value-weighted mean, and the median EBITDA multiples for 22 S&P industries. The range of these multiple estimates varies across industries. In some industries, such as chemicals and telephones, the four different measures are virtually identical. In other industries, such as computer hardware and metals mining, the differences across the multiples are substantial. For example, the value-weighted mean EBITDA multiple for computer hardware is 7.0 whereas the simple mean is 12.0, a difference of over 71%.

In evaluating the four alternative estimators, we use the Gibbs results in two ways. First, we show that the harmonic mean is most consistent with the estimated error structure from the Gibbs approach. Second, we use the multiples from the Gibbs approach as a minimum variance benchmark and evaluate the four alternative estimators relative to the Gibbs estimate. We compute the three types of means and the median for each of the 22 industries and compare them to the Gibbs estimate of the multiple. Our results show that the harmonic mean is the closest to the Gibbs estimate. Mathematically, the harmonic mean is always less than the simple mean. The closeness of the harmonic mean to the Gibbs estimate, therefore, implies that valuations based on the simple mean will consistently overestimate value.

In section 5 we examine three common measures of comparability: EBITDA, earnings before interest and taxes (EBIT), and revenue. We compute the harmonic means for each of our 22 industries for each of the three measures of comparability. We then compare the dispersion of the multiples to identify which measure of comparability results in the narrowest distribution. Although the results vary by industry, EBITDA appears to be the best single basis of substitutability for the industries we examine.

2. A Model of Multiples

In this section we provide a rationale for our econometric specification of the multiples model. The model, as typically used in practice is simple and straightforward: Value V equals the basis of substitutability X times the multiple M. That means there is a linear relation within a given industry between value and the basis of substitutability:

$$V_j = MX_j + \varepsilon_j \tag{3}$$

The difficulty in estimating (3) directly is that the valuation errors ε are unlikely to be independent of value because firms with higher values are likely to have larger errors. The multiple valuation model (3) can be interpreted as an application of a growing perpetuity of cash flows valuation model. The multiple in this situation depends on the relationship between the basis of substitutability and the expected cash flows, the discount rate, and the growth rate. Because errors arise in all three elements of the capitalization factor, the errors in (3) will depend on value.

The first source of error in a perpetuity model is in the relationship between the basis of substitutability and the expected cash flows. We define the expected cash flow one period hence *CF* for firm *j* as a multiple δ of the basis of substitutability *X* plus a firm specific error ε_1:

$$CF_j = \delta X_j + \varepsilon_{1j} \tag{4}$$

Errors also arise when the expected growth and discount rate for each firm *j* differ from the industry average rates:

$$r_j = r + \varepsilon_{2j} \text{ and} \tag{5}$$

$$g_j = g + \varepsilon_{3j}$$

The familiar perpetuity relationship relates value to cash flow, the discount rate, and the growth rate. We substitute (4) and (5) into this formula.

$$V_j = \frac{CF_j}{r_j - g_j} = \frac{\delta}{r-g} X_j + \frac{1}{r-g}\left[\varepsilon_{1j} + V_j\left(\varepsilon_{3j} - \varepsilon_{2j}\right)\right] \tag{6}$$

The errors in (6) depend on value, with value entering directly into the error term. The three separate sources of error may also depend on *V*. Consistent with errors depending on value, we assume that the errors in (3) are proportional to a power function of value:

$$\varepsilon_j \sim V_j^{\lambda} \tag{7}$$

The econometric challenge of estimating the multiple *M* is that the value of the firm *V* appears on both the left-hand side of the equation as the dependent variable and on the right-hand side in the error term. We therefore transform (3) into an econometric model in which the basis of comparability *X* appears on the left-hand side and value *V* appears on the right-hand side.

$$X_j = \frac{1}{M} V_j + e_j \tag{8}$$

$$E\left[e_j^2\right] = \sigma^2 V_j^{2\lambda}$$

where the new error e is equal to $\frac{1}{M}\varepsilon$.

Estimating multiples using (8) from comparable companies or transactions invariably involves a small number of observations.There are only 10 firms on average, for example, across the 22 S&P industries that we use in our empirical analysis. . As a result, relying on large sample econometric techniques is inappropriate. Finding minimum variance estimates for the parameters in (8) therefore requires a distributional assumption. Our assumption is that the errors *e* are normally distributed.

We test this assumption empirically. Our data on the S&P 500 for 1995 is from COMPUSTAT. Value *V* is the sum of the market value of equity, book value of long term debt, debt due in one year, and notes payable. For the basis of substitutability *X*, we use revenue, earnings before interest, taxes, and depreciation (EBITDA), and earnings before interest and tax (EBIT). Any firm without all seven of these items in the COMPUSTAT database for 1995 is excluded. S & P industry classifications divide the 500 firms into 103 groups. Only those industry groupings that contain at least seven firms are included. The final sample consists of 225 firms in 22 industry groups.

Figure 2 and Table 1 summarize our analysis on the normality of the errors from our econometric model. We calculate the residuals *e* from (8) using the harmonic mean as the multiple *M*. Because we find that the standard deviation of the errors scale linearly with value in Section 3, we scale *e* by *V* in the empirical tests of normality. Figure 2 presents a histogram of the scaled errors Figures 2a and 2b show that the EBITDA and EBIT errors appear to be normally distributed. Both histograms appear symmetric and roughly conform to the superimposed normal distribution. The revenue multiple, in contrast, does not appear to be normal because the distribution is skewed to the right.

Table 1 presents more formal test statistics for the normality of the errors. The table reports chi-squared tests of the null hypothesis that the error distributions are normal. We report the results for two statistical tests. The first panel of Table 1 presents separate tests for skewness and kurtosis and a combined test statistic. These tests are described in D'Agostino et al. (1990). The results support the casual observation of figure 2. For EBITDA, the combined test statistic is 1.99, and the associated p-value is 0.37. Because this p-value exceeds 5%, there is no evidence to reject the null hypothesis of normality. For EBIT, the p-value is 0.17, still well above the 5% cutoff. We reject the normality of the revenue errors with a p-value less than 0.01. Looking separately at skewness and kurtosis reveals that the revenue errors do not have fat tails, or kurtosis, but they are skewed: The p-value for kurtosis is 0.29, while the p-value for skewness is below 0.01.

The second panel presents Shapiro and Wilk (1965) tests of normality. These tests support the qualitative conclusions of the skewness-kurtosis test in panel A. For EBITDA, the chi-squared test statistic is 0.49, and the associated p-value is 0.31. The Shapiro-Wilk p-value is lower, but we are again unable to reject the null hypothesis that the EBITDA are normally distributed at the 5% level of significance. For EBIT, the p-value also lies above the 5% cutoff, but by a small margin. Finally, we again conclude that the revenue errors are not normally distributed at the 1% level.

These statistical tests suggest that our assumption of normality for the EBITDA and EBIT errors e in (8) is reasonable. Because the revenue errors appear not to be normally distributed, we view the Gibbs estimation of (8) for revenue as only an approximate solution.

3. Measurement of Multiples

This section describes the empirical estimation of the econometric model developed in section 2. In section 3.1, we describe the Gibbs sampling approach used to simultaneously estimate the multiple and the error structure. Section 3.2 presents the results from the Gibbs estimation.

3.1. The Econometric Approach

Gibbs sampling, a special case of Markov Chain Monte Carlo (MCMC) simulation, is a simple and powerful tool for estimation in small samplesWe begin by using Gibbs sampling to simultaneously estimate the multiple and the error specification. It is not practical to estimate a separate error structure for each sample of comparable firms. We therefore impose the restriction that λ, the coefficient of the power function that describes our errors, is constant across industries. We do however allow for a separate multiple M_i and a separate variance parameter σ_i for each industry. This approach gives us more degrees of freedom for estimating the form of the heteroskedasticity. Hence, the econometric specification for the model in (8) is:

$$X_{ij} = \frac{1}{M_i} V_{ij} + e_{ij} \tag{9a}$$

$$E\left[e_{ij}^2\right] = \sigma_i^2 V_{ij}^{2\lambda} \tag{9b}$$

where the subscript i denotes the industry and the subscript j denotes firms within the industry.

We estimate the industry multiples M and the variance parameters, λ and σ. With our industry definitions, estimating an industry multiple leaves as few as five degrees of freedom (seven comparable firms and two industry parameters, M and σ). For this reason, relying on large-sample, or asymptotic, econometric theory is inappropriate. The maximum likelihood (ML) approach therefore may not provide minimum variance estimates of the multiple. Even when the variance parameters λ and σ are known, only the ML estimate of the inverse multiple $\frac{1}{M}$ is guaranteed to be minimum variance in small samples.

Gibbs sampling is an alternative approach that provides minimum variance estimates of M, λ, and σ in a small sample. The procedure is summarized in figure 3. Before beginning the simulation, we choose initial values for λ and σ. We then compute the mean and variance of $\frac{1}{M}$ conditional on the initial values using regression analysis on (9a) and (9b). Because the errors e are normally distributed conditional on value V, $\frac{1}{M}$ is also distributed normally. We draw a value for $\frac{1}{M}$ at random from its normal distribution.

With the drawn $\frac{1}{M}$, we calculate the residuals e from (9a). The residuals can be used to calculate the mean and variance of λ conditional on the initial value of σ using regression analysis on a log transformation of (9b). The transformation removes the expectations operator and takes logs of both sides of the equation.

$$e_{ij}^2 = \sigma_i^2 V_{ij}^{2\lambda} u_{ij} \tag{10}$$

$$\ln e_{ij}^2 - \ln \sigma_i^2 = \lambda \ln V_{ij}^2 + \ln u_{ij}$$

Because we use the entire sample of 225 firms to estimate (10), λ is distributed approximately normally even though the errors may not be.[1] We draw a value for λ at random from its asymptotic normal distribution.

With the calculated residuals *e* and the drawn λ, we can transform (9b):

$$E\left[\frac{e_{ij}^2}{V_{ij}^{2\lambda}}\right] = \sigma_i^2 \qquad (11)$$

Because the scaled errors *e* are normally distributed, $\frac{1}{\sigma^2}$ has a gamma distribution. The parameters of the gamma distribution are a function of the squared residuals scaled by value as in (11) and the number of firms in industry *i*. We draw a value for $\frac{1}{\sigma^2}$ at random from its gamma distribution.

Now, we have drawn values of λ and σ. This means that we can draw a value of $\frac{1}{M}$ as before and repeat the process for a second iteration. After the first 100 iterations, the initial values become irrelevant.[2] The subsequent 1,000 iterations represent draws from the joint distribution of the parameters in (9). Because each draw has an equal probability, the average of 1,000 Gibbs draws provides minimum variance parameter estimates.

[1] We also include fixed industry effects in the estimation of (10) to ensure that λ reflects the influence of value on the errors within and not across industries.

[2] Gelman et al. (1995) describe the problem of assessing convergence in iterative simulation.

3.2. Empirical Results

In Table 2, we present the results from the Gibbs estimation of the empirical model in (9). The first two columns of Table 2 show the results using EBITDA data, the next two columns perform the same estimation for EBIT, and the last two columns report results for revenue. The parameter estimates are the average of 1,000 Gibbs draws from the joint distribution of the parameters in (9). The reported standard errors are the standard deviation of the 1,000 Gibbs draws.

Panel A of the table shows the minimum variance estimates and standard errors for the 22 industry multiples M. For EBITDA, the minimum variance multiples range from 4.6 for paper products to 19.0 for gold and precious metals mining firms. This means that an average paper products firm was worth 4.6 times EBITDA in 1995, while an average gold and precious metals mining firm was worth 19.0 times EBITDA. The standard errors range from 0.26 for telephone, or less than 4% of the multiple estimate of 6.7, to 6.21 for gold and precious metals, or over 32% of the multiple estimate of 19.0. For EBIT, which is always less than EBITDA, the minimum variance multiples are higher. The pattern across industries is similar however, ranging from 6.6 for paper products to 65.3 for gold and precious metals. Revenue, which is always higher than EBITDA, produces multiples that are lower than and less correlated with the EBITDA multiples. The lowest multiple is computer hardware at 0.8 and the highest is gold and precious metals at 4.9.

In panel B, we present the parameter estimates and standard errors for the common variance parameter λ. As with M, the estimates are equal to the average of 1,000 Gibbs draws for λ. The estimate for λ is equal to 0.99 for EBITDA, 0.88 for EBIT, and 0.99 for revenue. The standard deviations of 0.12 for EBITDA, 0.12 for EBIT, and 0.18 for revenue suggest that it is unlikely that λ could be less than 0.65, two standard deviations below the mean, or greater than 1.35, two standard deviations above the mean.

Because λ is not necessarily normally distributed, this may not be an appropriate test. As a check, we look at its full distribution. Figure 4 plots the results from the Gibbs simulation for λ. By grouping the draws together, we form a picture of the distribution of λ. The midpoint of each group of draws is reported on the horizontal axis. For example, the group labeled 1.0 contains draws lying between 0.975 and 1.025. We plot the number of draws in each group on the vertical axis. There are a total of 1,000 draws, so a level of 100 indicates that 10% of the distribution lies in that group. The overall picture confirms that λ most likely is between 0.65 and 1.35. For EBITDA, the draws are centered on 1.0. EBIT is centered a somewhat below 1.0 and revenue somewhat above.

4. Common Multiples

Although the Gibbs estimation provides a multiple for each industry, we view it as an impractical method of estimating multiples for practitioners. The Gibbs approach requires that the error structure and the multiples be estimated simultaneously and thereby requires data across industries. The typical practitioner problem, however, involves estimating a multiple for a single industry or group of transactions. We therefore also consider four common estimators of multiples: the median, the simple mean, the harmonic mean, and the value-weighted mean.

In section 4.1, we use the Gibbs point estimates of the variance parameter λ from section 3 to derive a maximum likelihood (ML) estimate of the multiple. Although ML does not necessarily provide minimum variance estimators even when λ is known, it does represent a simpler approach. In section 4.2, we empirically examine the four common multiples empirically and compare their performance against the Gibbs minimum variance estimates. The ML analysis and our empirical results both suggest that the harmonic mean is the best among common multiples.

4.1. Maximum Likelihood Estimation of Multiples

Our Gibbs estimates in section 3 show that λ, the coefficient of the power function that describes the errors in (8), is centered around one. When the variance parameter λ is equal to one, maximum likelihood estimation (ML) of *M* has a familiar solution. Because the error *e* in (8) is normally distributed, maximizing the likelihood function is also equivalent to minimizing the weighted sum of squared errors. Our empirical model, with λ equal to one, is:

$$X_j = \frac{1}{M} V_j + V_j v_j \tag{12}$$

where *v* is normally distributed with constant variance σ^2. Dividing both sides by value yields:

$$\frac{X_j}{V_j} = \frac{1}{M} + v_j \tag{13}$$

The least squares estimate of *M* in (13) is the harmonic mean:

$$1 \Big/ \sum_j \frac{X_j}{V_j} \tag{14}$$

The harmonic mean is therefore the ML estimate implied by the error structure estimated in section 3. In the next subsection, we compare the harmonic mean, along with the simple mean, the value-weighted mean, and the median, to the Gibbs minimum variance estimates.

4.2. Comparison of Common Multiples

Table 3 presents the four common multiples for the S&P 500 sample. The first column shows the number of firms in each industry. We consider only industries with at least seven firms. The largest industry, electric companies, has 26 firms. The second column calculates the simple mean, which is the average of the individual firm multiples $\frac{V}{X}$. For 1995, the mean industry multiple is between 4.8 for forest and paper products and 23.2 for gold and precious metals mining. The average of the simple means across the 22 industry groups is 9.5.

The third, fourth, and fifth columns present the harmonic mean, the value-weighted mean, and the median. The harmonic mean is the inverse of the average of the individual firm yields $\frac{X}{V}$. The value-weighted mean is the sum of firm values across firms within an industry $\sum V$ divided by the sum of the basis of substitutability $\sum X$. The median is simply the median of the individual firm multiples $\frac{V}{X}$. The pattern of multiples across industries is similar for each of the four common multiples. In all four cases, the paper products industry has the lowest multiple at around 4.8, and gold and precious metals mining has the highest multiple at about 18.0.

The last column of Table 3 shows the economic importance of multiple estimation. We calculate the range between the lowest and highest multiple as a fraction of the lowest multiple. For example, the maximum multiple for the auto parts industry is the simple mean of 7.0. The minimum multiple is the value-weighted mean of 6.0. The last column shows the maximum error from multiple estimation in percentage terms. This is the range of 1.0 divided by the minimum of 6.0, or about 17%. If the value-weighted mean were minimum variance, using the simple mean would result in a valuation error of 17%. The economic importance of multiple estimation varies across the 22 industries. For the telephone industry, multiple estimation is irrelevant: The maximum estimate is within 2% of the minimum. For computer hardware, in contrast, changing the approach can lead to as much as a 71% difference in the industry multiple.

We compare each of the four common multiples to the minimum variance multiples from the Gibbs estimationin table 4. The second column of table 4 reproduces the minimum variance multiple estimates from Table 2 for each of the 22 industry groups. In the next four columns, we evaluate the performance of the four common multiples. Our measure of performance is the absolute difference between the common multiple estimate and the minimum variance multiple expressed as a percentage of the minimum variance multiple. For example, the simple mean from Table 3 for banks is 6.2. This is 0.4 higher than the minimum variance estimate of 5.8. This difference is 6.17% of the minimum variance multiple.

The third column shows the performance of the simple mean. The simple mean is as much as 39% different from the minimum variance estimate. For some industries however, such as telephone companies, the difference is less than 1%. On average the difference is approximately 8.5%. The fourth column shows the performance of the harmonic mean. In contrast to the simple mean, the difference from the minimum variance estimate is never greater than about 7%, and the average is less than 2%. The fifth column shows the value-weighted mean, and the sixth column shows the median.

The average performance of the value-weighted mean is similar to the simple mean. The difference from the minimum variance estimate is as high as about 50% and averages about 10%. The median, which is commonly used in practice, performs better. The average error is around 6% and the worst difference is only 15%. The median however does fall short of the harmonic mean.

Table 4 shows that the harmonic mean empirically is very close to the minimum variance multiple from Table 2. This result is consistent with and closely related to the finding that the harmonic mean is the maximum likelihood estimate when, as the Gibbs results suggest, the standard deviation of the errors is proportional to value. These results suggest that the harmonic mean should be used when estimating a single industry multiple. The superiority of the harmonic mean is also economically reasonable. The harmonic mean effectively averages the yields, which are the inverse of the multiples. By averaging the yields, the harmonic mean gives equal weight to equal dollar investments. Because the simple mean is always greater than the harmonic mean, using the simple mean instead of the harmonic mean will consistently over-estimate value.[3]

5. Basis of Substitutability

We compute the harmonic mean multiples for each of our 22 industries using three common measures of comparability: EBITDA, EBIT, and revenue. We then compare the distribution of the multiples and identify which measure of comparability results in the narrowest distribution. We focus on the dispersion of the multiples for two reasons. First,

[3] This mathematical relationship holds whenever the individual firm multiples are all greater than zero.

economically, a narrow distribution of multiples for firms within an industry around the harmonic mean indicates a common value driver across firms in the industry. A substantial dispersion around the harmonic mean, in contrast, indicates that the basis of substitutability is not an effective descriptor of value. Second, statistically, the standard deviation of the harmonic mean measures the precision of the estimator. The lower the standard deviation, the more effectively the multiple method describes value. Thus, the best basis of substitutability is the choice that results in the lowest standard deviation around the harmonic mean.

The standard deviation of the harmonic mean is influenced by two factors: the number of comparable firms and the dispersion of the errors from (8). Holding dispersion constant, the number of comparable firms reduces the standard deviation. More firms mean more information about the industry multiple. Holding constant the number of comparable firms, the dispersion across firms increases the standard deviation. Technically, we define dispersion as the average squared error, where the error is equal to the difference between the individual firm yield $\frac{X}{V}$ and the average yield.

Because the number of comparable firms is constant within each industry, we use the dispersion of the errors to measure the accuracy of the harmonic mean for each of the three measures of comparability. The first set of two columns in Table 5 looks at EBITDA multiples. The first of the two columns shows the harmonic mean multiple and the second of the two columns shows the dispersion of the errors. To measure dispersion, we use the standard deviation of the individual firm yields $\frac{X}{V}$. To compare across the three measures of comparability, we scale this standard deviation by the average yield.

The second set of two columns performs the same set of calculations for EBIT, and the last set of two columns shows results for revenue. The final column reports which of the three basis of substitutability has the narrowest distribution measured by the standard deviation of the yields.

EBITDA is the best basis of substitutability for 10 of the 22 industries. The standard deviation of the yields ranges from about 10% of the industry average yield for telephone to 50% for gold and precious metals. The overall average is 28% and the median is 27%. If the yields are normally distributed, this means that about two thirds of the individual firm yields will lie within 28% of the average. EBIT, which is best for 9 of the 22 industries, performs almost as well as EBITDA. The standard deviations range from 8% to 161%. The overall average is 39%, but the median is only 29%. A simple t-test rejects the hypothesis that the mean EBIT and EBITDA standard deviations are the same at the 10% level of significance. Revenue multiples are worse. The standard deviations range from 12% to 120%. The overall average is 40% and the median is 35%. A t-test in this case rejects the hypothesis that the mean revenue and EBITDA standard deviations are the same at the 1% level of significance.

The basis of substitutability that provides the most precise estimate of value varies by industry because the underlying value drivers differ across industries. In some industries, for example, chemicals, value seems to be proportional to revenue, whereas in others EBITDA or EBIT proves the best basis of substitutability. Furthermore, while we have limited our analysis to financial measures, operating statistics like number of customers serviced may be better measures of substitutability. Our purpose in this section is twofold. First, we argue that the basis of substitutability should be selected by choosing the measure that minimizes the spread across yields within an industry. Second, we show that the basis varies across industries.

6. Conclusions

This paper focuses on two implementation challenges when using valuation multiples: how to estimate the industry multiple and how to choose a measure of financial performance as a basis of substitutability.

We use Gibbs sampling to simultaneously estimate minimum variance multiples and the error structure for 22 S&P industries in 1995. The estimated error structure is consistent with the harmonic mean, and the harmonic mean is the closest, out of four common multiple estimators, to the Gibbs sampling results. Thus, our answer to the first implementation problem – how to estimate the industry multiple – is to use the harmonic mean. Because the harmonic mean is mathematically always less than the simple mean, the results imply that using the simple mean industry multiple will overestimate value.

We argue that the basis of substitutability should be selected by choosing the measure that minimizes the spread across multiples within an industry. To study alternative bases of substitutability, we examine the harmonic mean multiple based on EBIT, EBITDA, and revenue. We show that the basis varies across industries. One explanation is that the basis of substitutability that provides the most precise estimate of value varies by industry because the underlying value drivers differ across industries.

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Figure 1. Mutliple measurement. EBITDA multiples for the S&P 500 in 1995. Industries are plotted on the horizontal axis. On the vertical axis, we shows the arithmetic mean, harmonic mean, value-weighted mean, and median of total firm value to EBITDA. The sample includes only those S&P 500 industry groups that contain at least seven firms.

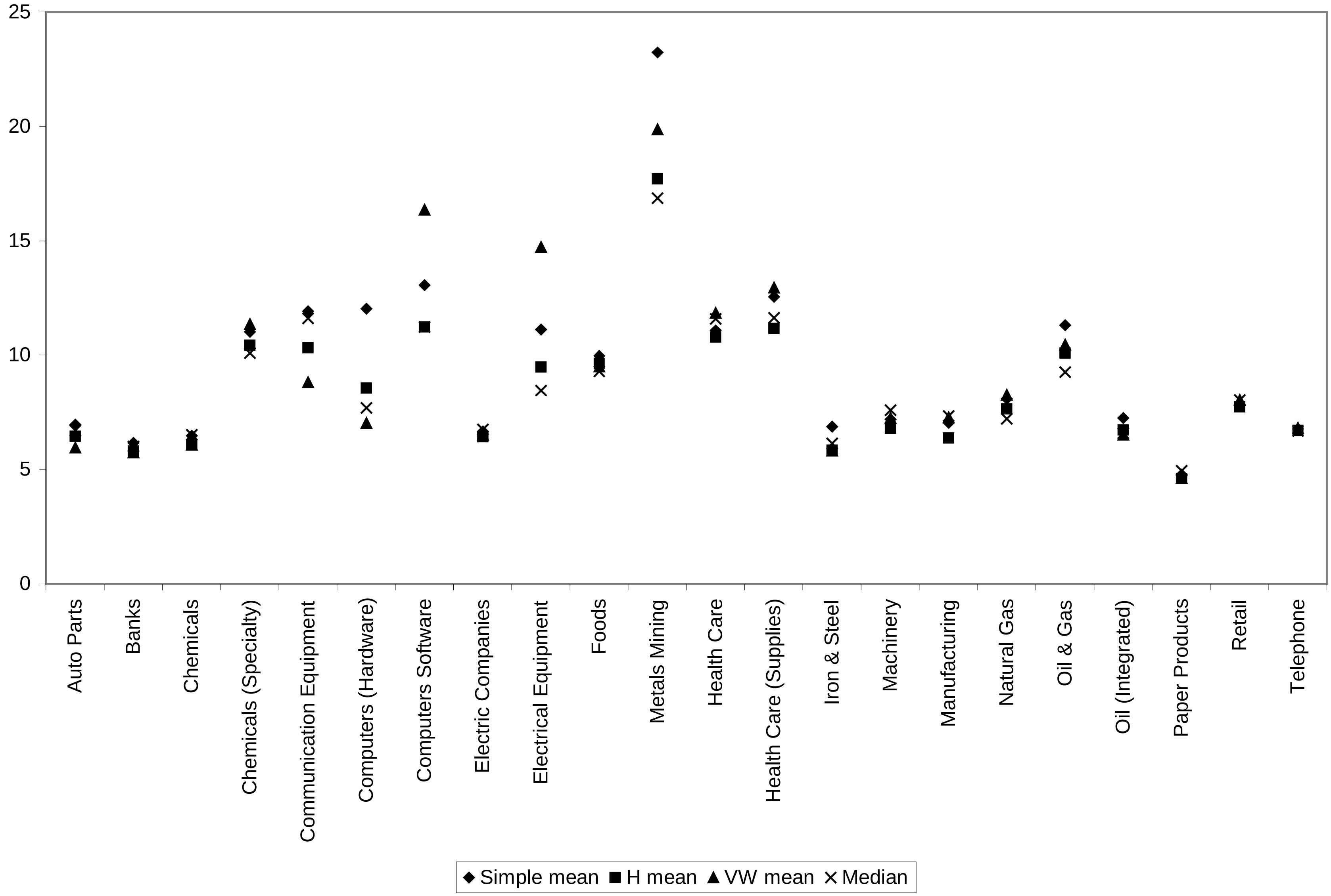

Figure 2. Normality assumption. Plot of the error term using the harmonic mean multiple. The errors are standardized to have zero mean and unit variance.

$$\text{Error} = \frac{1}{s_j}\left(\frac{X_{ij}}{V_{ij}} - \frac{1}{M_j}\right)$$

The distribution of the errors is plotted using data on EBITDA (1a), EBIT (1b), and revenue (1c). The sample includes only those S&P 500 industry groups that contain at least seven firms.

2a. EBITDA distribution

2b. EBIT distribution

2c. Revenue distribution

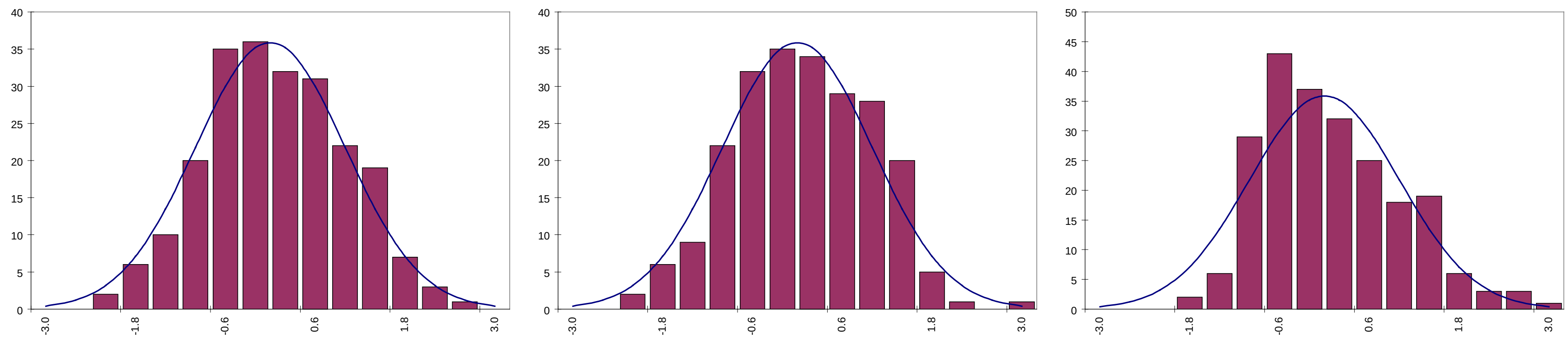

Figure 3. Gibbs sampling flow diagram. Flow diagram for a Gibbs sampling estimation of the following model of multiples:

$$X_{ij} = \frac{1}{M_i} V_{ij} + e_{ij} \qquad E\left[e_{ij}^2\right] = \sigma_i^2 V_{ij}^{2\lambda}$$

We iteratively draw values at random from the distribution of each parameter, *M*, λ, and σ, conditional on the data and the other parameter draws. In addition, we assume that the errors *e* are normally distributed conditional on *V*.

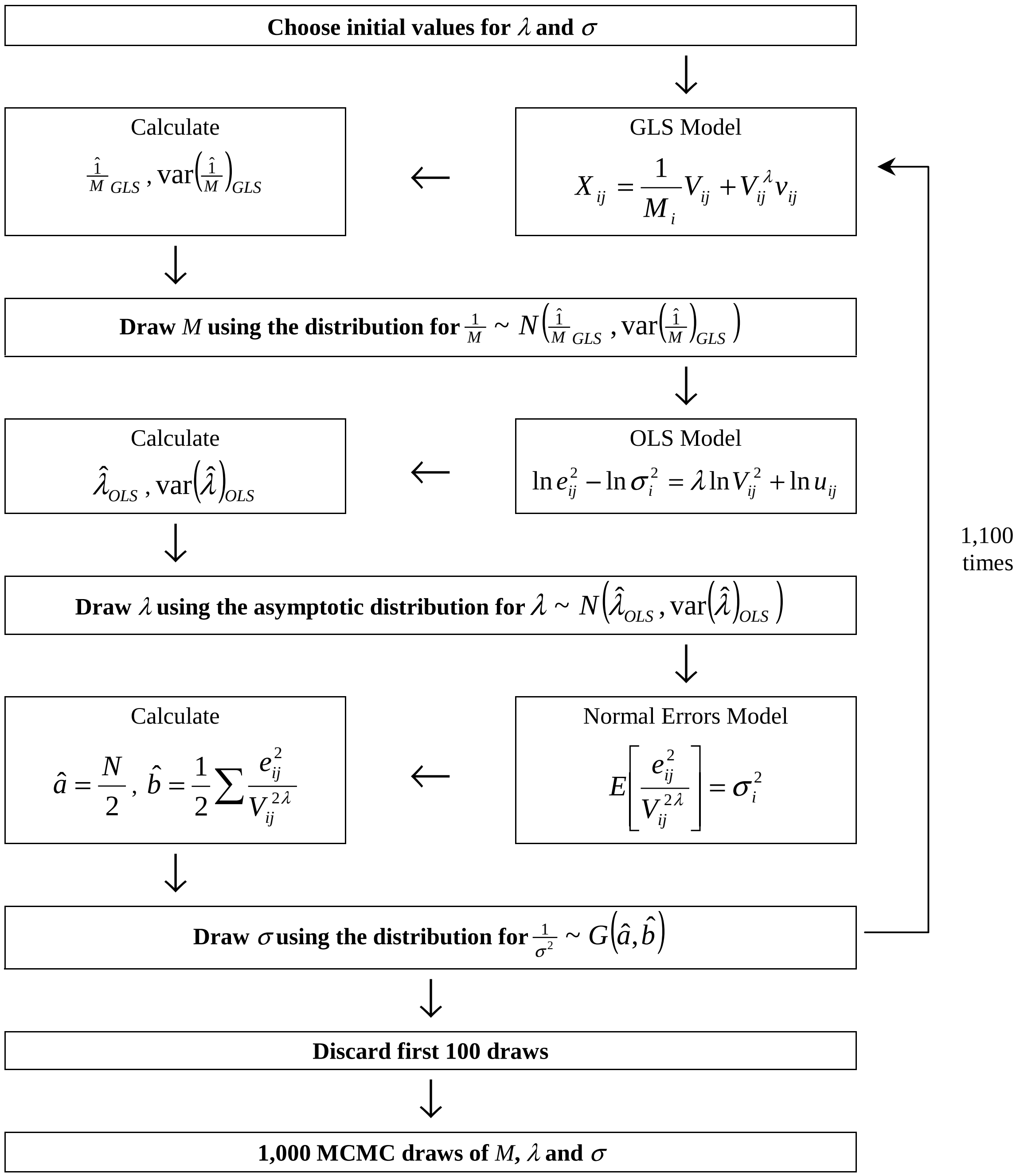

Figure 4. Empirical model of multiples. The posterior distribution of λ for a Gibbs sampling estimation of the following model of multiples:

$$X_{ij} = \frac{1}{M_i} V_{ij} + e_{ij} \qquad E\left[e_{ij}^2\right] = \sigma_i^2 V_{ij}^{2\lambda}$$

The distribution of the parameter estimate is plotted using data on EBITDA, EBIT, and revenue. The sample includes only those S&P 500 industry groups that contain at least seven firms.

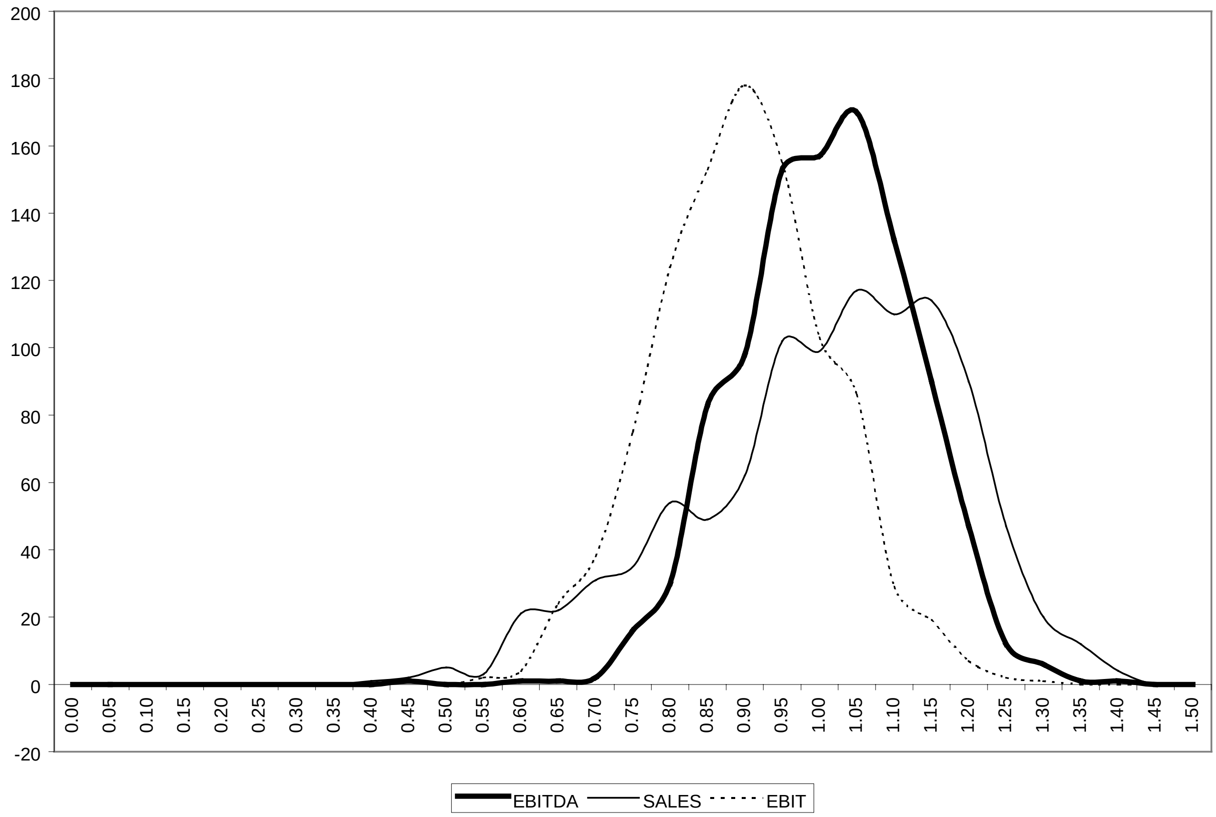

Table 1. Normality assumption. Tests of a normal error term using the harmonic mean multiple. The errors are standardized to have zero mean and unit variance.

$$\text{Error} = \frac{1}{s_j}\left(\frac{X_{ij}}{V_{ij}} - \frac{1}{M_j}\right)$$

The distribution of the errors is tested against the hypothesis of no skewness, no kurtosis, no skewness or kurtosis, and using the Shapiro-Wilk and Shapiro-Francia tests of normality. The tests use data on EBITDA, EBIT, and revenue. The sample includes only those S&P 500 industry groups that contain at least seven firms.

		EBITDA		*EBIT*		*Revenue*	
Industry	**N**	**χ^2 Test Statistic**	**p-value**	**χ^2 Test Statistic**	**p-value**	**χ^2 Test Statistic**	**p-value**
			Panel A: Skewness and Kurtosis				
Skewness-Kurtosis Test	225	1.99	[0.37]	3.49	[0.17]	14.20	[0.00]
No Skewness	225		[0.16]		[0.21]		[0.00]
No Kurtosis	225		[0.85]		[0.17]		[0.29]
			Panel B: Shapiro-Wilk Test				
Shapiro-Wilk Test	225	0.49	[0.31]	1.47	[0.07]	3.88	[0.00]

Table 2. Empirical model of multiples. Gibbs sampling estimation of the following empirical model of multiples:

$$X_{ij} = \frac{1}{M_i} V_{ij} + e_{ij} \qquad E\left[e_{ij}^2\right] = \sigma_i^2 V_{ij}^{2\lambda}$$

The parameter estimates and standard errors are calculated using data on EBITDA, EBIT, and revenue. Panel A shows parameter estimates and standard errors for the industry multiple *M*. Parameter estimates and standard errors for the common variance parameter λ are shown in panel B. The sample includes only those S&P 500 industry groups that contain at least seven firms.

	EBITDA		*EBIT*		*Revenue*	
Industry	**Mean**	**SE**	**Mean**	**SE**	**Mean**	**SE**
			Panel A: Industry Multiple *M*			
Auto Parts & Equipment	6.5	0.82	9.0	0.78	0.8	0.14
Banks (Major Regional)	5.8	0.31	6.8	0.42	1.7	0.09
Chemicals	6.2	0.76	8.9	1.44	1.4	0.13
Chemicals (Specialty)	10.6	1.25	14.0	1.80	2.3	0.30
Communication Equipment	10.6	2.16	14.6	1.99	1.7	0.54
Computers (Hardware)	8.7	1.27	14.8	14.60	0.8	0.15
Computers Software/Services	11.8	2.58	17.7	2.92	2.4	23.07
Electric Companies	6.5	0.27	9.3	0.41	2.3	0.15
Electrical Equipment	9.9	1.72	14.9	2.81	1.6	0.54
Foods	9.7	0.57	12.4	0.57	1.3	0.94
Gold/Precious Metals Mining	19.0	6.21	65.3	726.44	4.9	0.95
Health Care (Diversified)	10.9	1.17	13.9	0.93	2.5	0.47
Health Care (Med Prods/Sups)	11.4	1.55	15.7	1.48	2.7	0.72
Iron & Steel	6.1	1.39	9.5	1.96	0.7	2.40
Machinery (Diversified)	6.9	0.75	9.3	0.54	0.9	0.14
Manufacturing (Diversified)	6.5	0.80	9.3	1.56	0.9	0.16
Natural Gas	7.7	0.49	13.1	1.33	1.4	0.15
Oil & Gas (Drilling & Equip)	10.3	1.39	24.7	37.39	1.5	0.46
Oil (Domestic Integrated)	6.8	0.84	15.5	8.71	1.2	0.19
Paper and Forest Products	4.6	0.31	6.6	0.61	0.9	0.08
Retail (Department Stores)	7.8	0.40	10.7	0.58	0.9	0.11
Telephone	6.7	0.26	12.0	0.37	2.8	0.15
			Panel B: Variance Parameter λ			
λ	0.99	0.12	0.88	0.12	0.99	0.18

Table 3. Multiple measurement. EBITDA multiples for the S&P 500 in 1995. The first column shows the number of firms in each industry group. The next four columns show the arithmetic mean, harmonic mean, value-weighted mean, and median of total firm value to EBITDA. The sixth column reports the range of the four multiples as a percentage of the minimum multiple. The sample includes only those S&P 500 industry groups that contain at least seven firms.

		EBITDA Valuation Multiples				
Industry	**N**	**Mean**	**Harmonic Mean**	**Value Weighted Mean**	**Median**	**Range (%)**
Auto Parts & Equipment	8	7.0	6.5	6.0	6.7	16.73
Banks (Major Regional)	21	6.2	5.8	5.7	6.0	7.44
Chemicals	8	6.5	6.1	6.1	6.5	7.43
Chemicals (Specialty)	7	11.0	10.4	11.3	10.1	12.38
Communication Equipment	8	11.9	10.3	8.8	11.6	35.35
Computers (Hardware)	12	12.0	8.6	7.0	7.7	71.46
Computers Software/Services	7	13.0	11.2	16.4	11.2	45.80
Electric Companies	26	6.7	6.4	6.5	6.8	4.77
Electrical Equipment	9	11.1	9.5	14.7	8.4	74.27
Foods	11	10.0	9.6	9.5	9.3	7.24
Gold/Precious Metals Mining	7	23.2	17.7	19.9	16.9	37.83
Health Care (Diversified)	7	11.1	10.8	11.8	11.6	9.66
Health Care (Med Prods/Sups)	10	12.6	11.2	13.0	11.6	16.12
Iron & Steel	7	6.9	5.8	5.8	6.2	17.73
Machinery (Diversified)	10	7.2	6.8	7.3	7.6	11.70
Manufacturing (Diversified)	13	7.1	6.4	7.3	7.3	15.29
Natural Gas	14	8.1	7.6	8.3	7.2	14.77
Oil & Gas (Drilling & Equip)	7	11.3	10.1	10.5	9.2	22.20
Oil (Domestic Integrated)	7	7.2	6.7	6.5	6.6	10.99
Paper and Forest Products	11	4.8	4.6	4.6	4.9	7.39
Retail (Department Stores)	7	7.8	7.7	8.0	8.0	3.62
Telephone	8	6.8	6.7	6.8	6.7	1.89
Average		9.5	8.5	9.2	8.6	20.55
Minimum		4.8	4.6	4.6	4.9	1.89
Maximum		23.2	17.7	19.9	16.9	74.27

Table 4. Evaluating simple multiples. EBITDA multiple errors as a percentage of the minimum variance multiple. The first column shows the minimum variance multiple estimated with Gibbs sampling. The next four columns show the arithmetic mean, harmonic mean, value-weighted mean, and median multiple errors. The multiple error is defined as the absolute difference between the multiple estimate and the minimum variance multiple expressed as a percentage of the minimum variance multiple. The sample includes only those S&P 500 industry groups that contain at least seven firms.

			EBITDA Multiple Errors (%)			
Industry	**N**	**Minimum Variance Multiple**	**Mean**	**Harmonic Mean**	**Value Weighted Mean**	**Median**
Auto Parts & Equipment	8	6.5	6.25	-1.43	-8.98	2.40
Banks (Major Regional)	21	5.8	6.17	-0.13	-1.18	3.75
Chemicals	8	6.2	5.25	-1.14	-1.39	5.94
Chemicals (Specialty)	7	10.6	3.57	-1.95	6.61	-5.14
Communication Equipment	8	10.6	12.43	-2.58	-16.93	9.49
Computers (Hardware)	12	8.7	38.81	-1.35	-19.04	-11.14
Computers Software/Services	7	11.8	10.26	-5.06	38.25	-5.18
Electric Companies	26	6.5	3.04	-0.24	0.78	4.52
Electrical Equipment	9	9.9	12.56	-4.16	48.87	-14.58
Foods	11	9.7	2.94	-0.56	-1.94	-4.02
Gold/Precious Metals Mining	7	19.0	22.13	-6.96	4.38	-11.39
Health Care (Diversified)	7	10.9	1.33	-1.25	8.29	5.98
Health Care (Med Prods/Sups)	10	11.4	10.17	-2.05	13.73	2.00
Iron & Steel	7	6.1	12.78	-4.04	-4.20	0.99
Machinery (Diversified)	10	6.9	4.71	-1.14	5.71	10.43
Manufacturing (Diversified)	13	6.5	8.81	-1.67	12.34	13.36
Natural Gas	14	7.7	5.25	-0.60	7.54	-6.30
Oil & Gas (Drilling & Equip)	7	10.3	9.92	-1.83	1.66	-10.05
Oil (Domestic Integrated)	7	6.8	5.83	-1.70	-4.65	-3.80
Paper and Forest Products	11	4.6	3.16	-0.38	0.24	6.98
Retail (Department Stores)	7	7.8	0.53	-0.40	3.20	3.20
Telephone	8	6.7	0.71	-0.22	1.10	-0.77
Mean Error			8.48	-1.86	4.29	-0.15
Minimum			0.53	-6.96	-19.04	-14.58
Maximum			38.81	-0.13	48.87	13.36

Table 5. Accuracy of the harmonic mean multiple. Gibbs sampling estimates of the mean and dispersion of the harmonic mean multiple, using EBITDA, EBIT, and revenue. The first column shows the number of firms in each industry group. The next three sets of columns show the harmonic mean multiple and the standard deviation of the industry yields expressed as a percentage of average industry yield. The final column reports the best basis of substitutability for each industry using standard deviation as a criterion. The sample includes only those S&P 500 industry groups that contain at least seven firms.

		EBITDA		*EBIT*		*Revenue*		
Industry	**N**	**Harmonic Mean**	**Yield Standard Deviation (%)**	**Harmonic Mean**	**Yield Standard Deviation (%)**	**Harmonic Mean**	**Yield Standard Deviation (%)**	**Best Basis**
Auto Parts & Equipment	8	6.5	29.42	9.1	18.87	0.8	38.09	EBIT
Banks (Major Regional)	21	5.8	23.81	6.8	26.09	1.7	24.07	EBITDA
Chemicals	8	6.1	26.56	8.7	34.33	1.4	22.85	Revenue
Chemicals (Specialty)	7	10.4	24.33	13.6	26.52	2.2	26.66	EBITDA
Communication Equipment	8	10.3	40.81	14.6	32.79	1.6	58.92	EBIT
Computers (Hardware)	12	8.6	41.85	16.1	112.19	0.7	41.46	Revenue
Computers Software/Services	7	11.2	37.26	16.4	30.28	1.6	119.64	EBIT
Electric Companies	26	6.4	19.74	9.3	21.46	2.3	30.75	EBITDA
Electrical Equipment	9	9.5	32.49	13.3	33.82	1.4	35.73	EBITDA
Foods	11	9.6	17.09	12.4	13.99	1.3	57.61	EBIT
Gold/Precious Metals Mining	7	17.7	50.05	64.6	161.40	4.7	37.10	Revenue
Health Care (Diversified)	7	10.8	19.24	13.6	14.98	2.5	31.19	EBIT
Health Care (Med Prods/Sups)	10	11.2	33.64	15.1	23.38	2.5	52.35	EBIT
Iron & Steel	7	5.8	39.74	9.2	35.22	0.7	42.95	EBIT
Machinery (Diversified)	10	6.8	28.39	9.2	19.22	0.9	35.22	EBIT
Manufacturing (Diversified)	13	6.4	38.27	8.7	47.94	0.8	48.46	EBITDA
Natural Gas	14	7.6	20.98	12.7	33.61	1.3	34.00	EBITDA
Oil & Gas (Drilling & Equip)	7	10.1	29.31	22.9	72.12	1.4	45.07	EBITDA
Oil (Domestic Integrated)	7	6.7	24.55	15.1	59.05	1.2	30.35	EBITDA
Paper and Forest Products	11	4.6	19.85	6.6	28.18	0.9	24.90	EBITDA
Retail (Department Stores)	7	7.7	10.72	10.7	12.52	0.9	24.42	EBITDA
Telephone	8	6.7	10.43	11.9	7.63	2.8	11.57	EBIT