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Abstract

We study the good- and bad-times performance of equity portfolios formed on characteristics. Many characteristics associated with good performance during bad times – value, profitability, small size, safety, and total volatility – also perform well during good times. Stocks with characteristics signifying high liquidity, such as high turnover and low bid ask spreads, perform well during bad times but otherwise underperform. We develop a simple but flexible procedure to recover a "risk neutral alpha" that recognizes a 1% return experienced during bad times as being more valuable than a 1% return generated during good times. We also show how an investor can build a "rainy day" portfolio that minimizes underperformance during bad times.

One of the organizing principles of modern asset pricing theory is that investors should pay higher prices for securities that pay handsomely in bad times. In the Consumption CAPM, for example, the price of a risky asset, p_t, is determined by the future payoff x_{t+1}, times the rate of time preference, θ, and the marginal utility of consumption in the period in which the asset pays off:

$$p_t = E_t \left[\theta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right] \tag{1}$$

An equivalent formulation of this idea is that securities that perform well during bad times should earn low average returns.

However appealing the idea encapsulated by equation (1) may be, it has been of limited use in explaining the pricing of individual stocks. The simplest measure of a security's downside exposure – its CAPM β – has long been known to be insufficient to explain the cross-section of returns (Black, Jensen and Scholes (1972), Brennan (1971), Fama and French (1992)). Measures of downside risk based on consumption data have an even poorer record matching the data (Hanson and Singleton 1982, 1983; Parker 2003; Campbell and Cochrane 2000). Although some researchers have identified measures of downside exposure that correlate somewhat better with expected returns, the overall empirical success of the consumption CAPM for the cross-section of stocks has been limited.[1]

The Consumption CAPM still, however, serves an important normative purpose: investors *should* avoid securities that pay off poorly in bad states of the world, and even more so if those securities do not handsomely reward investors during ordinary times. In this paper, we embrace

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[1] Researchers have had slight success when measuring the correlation of stocks with longer-term consumption growth. However, the resulting measures of bad times do not easily conform to conventional views of what an investor would call bad times (Parker 2001; Bansal and Yaron 2004; Parker and Julliard 2005; Malloy, Moskowitz, Vissing-Jorgensen 2009 and many others).

this normative objective. We ask, for an equity investor who is worried about bad times in the future, what securities should she optimally hold? And, if this investor holds stocks that are less exposed to bad times, how much performance drag can such an investor expect? We answer these questions by adopting a two-state asset pricing model in which times are either good or bad. We show that adopting this simple approach recasts the performance of many well-known risk factors in a way that is useful for real-world investors.

Our analysis proceeds in three steps. First, we must settle on a definition of bad times. The concept of bad times is highly model- and preference-dependent. We adopt the approach of classifying as bad times periods that would be classified as bad by nearly any sensible representation of the stochastic discount factor, and would likely be considered as difficult periods by most investors. Specifically, most of our analysis is performed using a measure that captures periods of extended stock market drawdown occurring simultaneously with poor economic data. According to our measure, bad times are quarters with yearly excess market return of minus nineteen percent, negative GDP and earnings growth, and high inflation and unemployment. While our two state approach is a simplification of a continuum of states, it is easy to implement because one does not have to calibrate a utility function.

Second, we identify which types of stocks performed the best during bad times – professional investors sometimes refer to these stocks as "defensive equities." We sort stocks into quintiles based on their total ex post returns during bad times, and measure the characteristics of the best performing quintile at the start of each bad times episode. A number of distinct patterns emerge. Defensive stocks score high on value, profitability, "safety", are growing slowly, have low volatility and are highly liquid (high turnover and low bid ask spreads). These results only partly reflect the fact that some of these characteristics are associated with low CAPM betas.

Third, guided by the results above, we build long-short portfolios based on characteristics so as to measure the discount or premium associated with performing well during bad times. For each of these portfolios, we measure its unconditional market beta, and its conditional good-times and bad-times alpha. For example, the value factor HML has a CAPM beta of -0.06, a good-times alpha of 0.32% per month, and a bad times alpha of 1.09% per month. Having *both* a positive bad-times alpha and positive good-times alpha is suprising, but appears in many of the characteristics-based portfolios that we study.

How should an investor assess the state contingent bad times alpha achieved by these strategies? We propose a simple approach guided by the normative logic of the consumption CAPM and the pricing of the overall stock market. Namely, if the aggregate stock market is priced correctly on average, then we can back out the implicit value that investors place on achieving returns during good and bad times. We then use the implied state prices during good and bad states to evaluate the realized returns of our characteristic-sorted portfolios. The difference between the state-price weighted returns across good and bad states yields each strategy's "risk neutral alpha." Our measure is akin to evaluating equity portfolio returns under the risk neutral measure. Our approach differs from the more standard CAPM alpha, which after controlling for beta, equally rewards performance in good and bad times. For example, the CAPM implicitly puts the same value on a loss in December 2008 and a loss in January 2004—both months in which the market was up by 2.2 percent.

Our analysis yields the following conclusions. Portfolios based on liquidity measures outperform during bad times but underperform during good times, thus achieving risk neutral alphas close to zero. For the characteristics of value, and profitability, however, the risk neutral alphas are even higher than the CAPM alphas, reflecting that these strategies perform even better

during bad times than during good times. Finally, low-risk factors have bad-times alphas around zero, but since they have large good-times alphas they have positive and large risk neutral alphas. We combine these insights to construct a "rainy day portfolio" that is long stocks that perform well during bad times: those with high book-to-market, high profitability, and market capitalization below the median. The rainy day portfolio has an unconditional CAPM alpha of .8 percent per month and it has a bad times alpha of 2.3 percent per month.

We don't mean to suggest that our characteristics-based portfolios are the only ways that an investor can immunize herself against bad times. For example, the most well-known bad-time protection strategy is to buy out of the money puts on the market index. Applying our two-state methodology to the profits from this options-based strategy, we show that such a strategy has a positive good-times alpha and a negative bad times alpha, with a risk neutral alpha close to zero. This lends some confidence to our classification of good and bad times.

In the last section, we describe a number of extensions and robustness tests. We show that industries can be easily classified according to their exposure to good and bad times; however, defensive industries tend to underperform slightly during good times—this effect is subtle as the correlation between good- and bad-times alpha is only -0.15. We show that the characteristics that we identify as being robust to bad times among US stocks (size, book-to-market, and profitability) have similar performance amongst international stocks as well. We also compute risk neutral alphas using different definitions of bad times and find that our results are largely robust across the different definitions.

A large literature in asset pricing connects measures of risk to expected returns of individual stocks or portfolios. A common starting point for many of these papers is the

observation that the securities market line – the relationship between average returns and the CAPM beta – is too flat relative to CAPM theory. A number of papers have developed more nuanced risk measures to fit the cross-section of returns, starting with Chen, Roll, and Ross's (1986) study of exposure to macroeconomic variables. Other studies include Ang, Chen, and Xing (2006) who analyze stocks' downside beta, and Parker (2003), Parker and Julliard (2005), Malloy, Moskowitz, and Vissing-Jorgensen (2009), and Yogo (2006) who study exposure to changes in consumption.[2] Our normative approach departs from the approach taken in most of the literature, because we do not attempt to fit any particular asset pricing model.

In Section I, we develop simple time-series measures of bad times. In Section II, we describe the ex-post characteristics of stocks that exhibited the best performance during bad times. Section III develops a simple procedure to risk adjust portfolio returns for their exposure to bad times and applies it to characteristic-sorted portfolios in US data 1963-2015. We also develop a "rainy day" portfolio built to minimize drawdowns in bad times. Section IV describes a number of extensions. Section V concludes.

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[2] For similar approaches, see also Jagannathan and Wang (1996); Heaton and Lucas (2000); Cochrane (1996); Vassalou (2003); Li, Vassalou, and Xing (2006); Ait-Sahalia, Parker, and Yogo (2004); Bansal, Dittmar, and Lundblad (2005); Hansen, Heaton, and Li (2006); Campbell and Vuolteenaho (2004). Lewellen, Nagel, and Shanken (2010) and Daniel and Titman (2012) offer a skeptical view on these approaches.

I. Identifying Bad Times

Bad times are periods where investors place a high value on investment payoffs. Asset pricing has admitted several definitions of bad times, which depend on the specification of investors' utility function. Intuitively, bad times may be when the economy is troubled and investors face a higher risk of job loss or reduced wages and have to reduce consumption as a result, or it may be when financial markets are troubled and investors have realized large financial losses. In the Sharpe-Lintner CAPM, states of the world are fully summarized by the return on the aggregate financial wealth portfolio. Consumption based models suggest that bad times are periods where the economy is troubled, or aggregate consumption has fallen (Lucas (1978), Breeden (1979) Campbell and Cochrane (2000)). To encompass the largest class of theoretical models, our baseline measure of bad times is when the economy is experiencing *both* economic and financial bad times.

We define *financial bad times* as quarters in which either the quarterly or the yearly US stock market excess return is in the bottom quintile of our sample, which runs from 1963 to 2013. By evaluating yearly excess returns as well as shorter-term quarterly returns, we ensure that we identify as bad times periods when the market is in a large drawdown, even if there has been a partial recovery. Specifically, if the market drops dramatically in one quarter, and neither recovers nor worsens in the following quarter, then this following quarter is likely to be defined as financial bad times despite having neither high nor low returns itself.

To define *economic bad times*, we use the NBER business cycle indicator. We define a quarter to be economic bad times if at least one of its months is registered as a recession by the

NBER. Our resulting classification of quarters into economic bad times is similar if we instead use industrial production or annual real GDP growth and select the bottom quintile in the time series.

We define the rainy day index as a quarterly dummy variable equal to one if the quarter has *both* economic and financial bad times and zero otherwise. While this restrictive definition may leave out some periods that would be admitted as a bad time under some specifications, our approach minimizes false positives. Figure 1 shows the time-series of the rainy day index.

Table 1 reports sample average of different economic and financial variables under different definitions of bad times. The first column shows the economic and financial features of our entire sample, and the remaining columns show the averages during different definitions of bad times. Rainy days are worse than normal times on nearly all dimensions: annualized excess market returns of minus nineteen percent, year-on-year GDP growth of minus one percent, earnings growth of minus twenty-five percent, higher than average inflation, higher than average unemployment, and a five percent yearly drop in home prices. Rainy days are also associated with a higher TED spread, a higher market volatility as proxied by the VIX, and more noise in the term structure as captured in Hu, Pan, and Wang's (2013) term structure noise measure.[3] Rainy days indeed appear to capture difficult times for almost any investor. Our rainy day index is strongly negatively correlated with growth of durable consumption, but uncorrelated with nondurable consumption.

We noted that our measure of financial bad times includes some of the rebound period after a stock market crash, when the market is still down significantly from peak. Table 1 also describes

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[3] TED spreads, VIX, and the noise measure are not available for the complete time-series.

a measure of bad times called "drawdown" that excludes the rebound period.[4] "Drawdown" bad times are quite similar to our baseline measure of financial bad times, however the market returns are somewhat lower.

Finally, while most of our discussion will center on the performance of US stocks during bad times, as a robustness check, we also perform our analysis on a global sample of stocks from 1986-2013. We construct a separate rainy day index for the global sample. We use the excess return to the global value-weighted market portfolio to construct our global financial bad times. We define global economic bad times as the periods where the growth in GDP for OECD countries is in the bottom quintile of the sample. As presented in Table 1, the features of the global rainy days are quite similar to the U.S. rainy days. Further, the correlation between the global rainy days and the U.S. rainy days is 0.82.

II.   Which Stocks Outperform during Bad Times?

To identify rainy day stocks, we first sort stocks into portfolios based on their ex-post performance during rainy days. We then calculate the ex ante characteristics of the portfolios and look for relationship between characteristics and returns.

We include the following characteristics in our analysis: market value, book-to-market, momentum, profitability, "growth", "safety", total volatility, turnover, modified Amihud ratio[5], and the bid-ask spread. As our profitability, growth, and safety measures we use the composite

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[4] The column labelled "drawdown" considers all market drawdowns with peak below minus ten percent: bad times are defined as the period between the beginning and the maximum of the drawdown period. We set the maximum length of a drawdown to be five years.

[5] We estimate Amihud ratios as suggested by Amihud (2002) except that we base our measures on turnover rather than raw volume, so as not to mechanically confound liquidity with size.

variables defined by Asness, Frazzini, and Pedersen (2014).[6] We update accounting information each June using information from the end of December in the previous year. We update market capitalization at the end of each month. Our choice of characteristics is guided by prior research. Characteristics can loosely be grouped into those related to fundamentals (size, profitability, safety and total volatility, and growth), past price movements (momentum), and liquidity factors (Amihud ratios, turnover, and bid-ask spreads). Many of these characteristics are known to produce unconditional alphas, but this is not a prerequisite for inclusion. Our primary focus here is on *when* these alphas are produced.

We group consecutive quarters of bad times into a total of seven periods (1969Q4-1970Q3; 1973Q4-1975Q1; 1980Q4; 1981Q3-1982Q2; 1990Q3-Q4; 2001Q1-Q4; 2008Q1-2009Q2). For each of the seven periods, we sort the stocks into quintiles based on the ex post return over the full bad times period, where the breakpoints are based on the ex post return to NYSE firms. Within each quintile, we calculate the equal weighted ex ante characteristics of each portfolio. This procedure results in seven periods with five portfolios in each. Our final characteristics estimate for each of the five portfolios is the average across the seven periods.[7] We measure characteristics as the cross-sectional percentile.

Panel A of Table 2 shows these results. The best performing quintile of stocks disproportionately contains value, safety, low growth, low total volatility, high Amihud ratios (i.e., illiquid), and low turnover. These results reflect, at least in part, the fact that some of the characteristics we examine are correlated with CAPM betas. Thus, in Panel B we instead sort

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[6] We estimate the modified Amihud ratio, turnover, and bid-ask spreads using daily data over 90 days. We require at least 30 observations to construct the characteristic. Bid-ask spreads are closing spreads which are available on CRSP from 1982 for NASDAQ firms and 1992 for NYSE/AMEX firms.

[7] The time-series weights are the number of quarters each consecutive period of rainy days last.

stocks on CAPM alphas realized during bad times. For each period, we compute alphas as the cumulative excess return to the stock minus its ex ante beta times the cumulative excess return to the market. We estimate the betas in the same way as Frazzini and Pedersen (2014).[8] Adjusting for market beta has the effect of reducing the overrepresentation of growth and total volatility among the groups of the best performing stocks. Adjusting for market beta also changes the conclusion about the liquidity characteristics, as the best performing stocks are now associated with high liquidity (high turnover and low bid ask spreads). Aside from that, the characteristics that perform well during bad times in Panel A seem to also be associated with CAPM alpha.

While these results suggest that equity investors may be able to hedge against bad times by holdings stocks with particular characteristics, the results are ex post and thus do not provide much quantitative guidance as to how effectively an investor could protect himself against bad times ex ante, and the results do not offer insight into the price investors must pay during good times for this bad times protection. We present formal tests in the next section.

III. Measuring the Performance of Rainy Day Stocks

We now describe the returns of portfolios based on the characteristics in Section II. We follow Fama and French and construct our factors as the intersection of six value-weighted portfolios formed on size and the portfolio's characteristic.[9] The turnover factor, for instance, is

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[8] We shrink the estimated betas towards their cross-sectional mean, $\hat{\beta}_i = w_i \hat{\beta}_i^{TS} + (1 - w_i)\beta^{XS}$, with shrinkage factor $w_i = 0.6$ and cross-sectional mean $\beta^{XS} = 1$. Our estimates of betas are given by $\hat{\beta}_i^{TS} = \hat{\rho}_{i,m} \frac{\hat{\sigma}_i}{\hat{\sigma}_m}$, where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ are the estimated volatilities of stock i and the market and $\hat{\rho}_{i,m}$ is their correlation. We use five-year rolling windows and overlapping three-day log returns, $r_{i,t}^{3d} = \sum_{k=0}^{2} \ln(1 + r_{t+k}^i)$, to estimate correlation. We use one-year rolling windows and one-day log returns to estimate volatilities. We require at least 750 trading days of non-missing return data to estimate correlation and 120 trading days of non-missing return data to estimate volatility.
[9] We use the size and momentum factors from Fama and French, the value factor from Asness and Frazzini (2013), and the safety, profitability, and growth factor from Asness, Frazzini, and Pedersen (2013).

long half a dollar in each of the two high turnover portfolios and short half a dollar in each of the two low turnover portfolios. We use conditional sorts for the portfolio breakpoints. We follow the convention that the long side of the portfolio buys the stocks that have high unconditional CAPM alpha, which is not necessarily the same as having a high alpha during bad times.

The first two rows of Panel A of Table 3 summarize average excess monthly returns during good and bad times. Value, profitability, safety, total volatility, and turnover-sorted portfolios outperform by more than 50 basis points per month during bad times, although the performance is only statistically significant in the case of value and profitability.

As in Section II, the results on excess returns partially reflect differential exposure of these long short portfolios to the stock market. To control for this exposure, we regress portfolio excess returns $r_t^{p,e}$ on the aggregate stock market excess return and a bad times dummy:

$$ r_t^{p,e} = \alpha_u + \alpha_b 1_b + \beta(r_t^m - r^f) + u_t \qquad (2) $$

Notice that we estimate a single beta in equation rather than separate good and bad times betas. The idea behind a constant beta is that it captures a simple market hedge achievable ex ante, even if bad times are not forecastable.[10]

Equation (2) measures the conditional performance of an investor who shorts β units of the market as a hedge. The return this investor achieves during good times is estimated as α_u, while the bad times performance is given by the sum $\alpha_u + \alpha_b$. Panel B of Table 3 shows these results. Column (1) shows that the Fama-French SMB portfolio, that buys small stocks and sells short

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[10] If states are defined only by market returns, then the statement that a portfolio has a high bad times alpha is equivalent to saying that it has a higher downside beta. This no longer holds exactly when bad times are defined based on variables other than the market return.

large stocks, has a good times alpha of 0.18 percentage points per month, and a beta of 0.19. In addition, small stocks earn an additional 0.27 percentage points per month in alpha during bad times, thus totaling 0.45 percentage points per month.

The value portfolio has a good times alpha of 0.32 percentage points per month, and a bad times alpha of 1.09 percentage points per month. Profitable stocks have a good times alpha of 0.25 basis points per month, and a bad times total alpha of 0.43 percentage points. Growth stocks, however, outperform during good times but underperform during bad times. "Safe" and low volatility stocks outperform (on a beta adjusted basis) during good times, and underperform modestly during bad times. Momentum exhibits enormous outperformance during good times (0.89 percentage points per month) but underperforms during bad times. The various characteristics measuring illiquidity – Amihud, low turnover and high bid-ask – all outperform during good times but underperform during bad times.

The main takeaway from this table is that for many characteristics that are associated with an unconditional alpha, this does not come at the expense of performance during bad times. Similarly, outperformance during bad times is not associated with any discount in good times. There are some characteristics, however, for which outperformance in bad times is associated with underperformance during good times. In particular, illiquid portfolios (high Amihud ratios, low turnover, high bid-ask spreads) all earn an alpha during good times, but pay for it with underperformance during bad times. Low turnover stocks, for example, achieve a good times alpha of 0.40 percentage points per month, but bad times performance is -0.22 percentage points per month.

Computing Risk Adjusted Performance

Although our conditional bad times alpha estimates are revealing, their magnitude is difficult to evaluate without a benchmark of how much outperformance in bad times should be equated with underperformance in good times (or conversely). For example, columns (5) and (7) show that safety and momentum underperform slightly during bad times, but on the other hand they experience large outperformance during good times.

Consider the stochastic discount factor representation of asset prices. For any portfolio p that is correctly priced:

$$0 = E\left[mr^{p,\mathrm{e}} \right] \tag{3}$$

in which and m denotes the stochastic discount factor, and $r^{p,\mathrm{e}}$ is the return to portfolio p net of the risk free rate. If there are only two states good (g) and bad (b), it then follows that:

$$\frac{m_b}{m_g} = -\frac{\pi_g}{\pi_b} \cdot \frac{r_g^{m,\mathrm{e}}}{r_b^{m,\mathrm{e}}}. \tag{4}$$

where π_s denotes the probability of state s.

To go from a ratio of state prices to the level, we make use of the fact that

$$1/(1+r^f) = E[m] = m_b \pi_b + m_g \pi_g. \tag{5}$$

This allows us to solve for state prices

$$m_b = \frac{1/(1+r^f)}{\pi_b - \dfrac{r_b^{m,e}}{r_g^{m,e}}\pi_b}$$

$$m_g = \frac{1/(1+r^f)}{\pi_g - \dfrac{r_g^{m,e}}{r_b^{m,e}}\pi_g}$$

(6)

We define the risk-neutral alpha for a portfolio p as

$$\alpha^{p,\mathrm{m}} \equiv \frac{E[mr^{p,e}]}{E[m]}.$$

(7)

which is simply the return evaluated under the risk neutral measure. In Appendix 1 we show how for any partition of outcomes into S states, we can compute a risk neutral alpha for any portfolio p, provided we have expected returns on S-1 other non-redundant assets.

Equation (7) can be implemented conditionally or unconditionally. We implement it unconditionally and back out state prices using the average performance of the market during good and bad times, a procedure that resembles Hansen and Jaganathan (1991). We start by observing that in the data, bad times occur in 13.2% of quarters with an average market return during these months of -1.72%. We can then back out the state prices using equation (6) and the average performance of the market during good and bad times. Under our definition of bad times, the risk neutral alpha of any portfolio works out to be approximately 0.33 times its excess return in bad times (even though bad times only occur 13.2% of the time) plus 0.67 times its excess returns in good times.

An attractive feature of this methodology is that the risk neutral alpha can be computed using state contingent excess returns, or using state contingent CAPM alphas, yielding an identical result. To see this, note that we can always write the excess return of portfolio p in state s as

$$r_s^{p,\text{excess}} = \alpha_s + \beta \cdot r_s^{m,\text{e}} \tag{8}$$

where β is the unconditional CAPM beta. But the risk neutral alpha of the market is zero, meaning we can use the alphas directly. The key to this is the use of the market as a reference portfolio for which the risk neutral alpha of zero is zero by assumption.

More generally, our identification of the risk neutral alpha relies on three assumptions. First, our two states must correctly identify all states of the world. Second, the value of the stochastic discount factor in each of the two states must be constant over time. Third, the probabilities for the two states must be constant over time. The second and third assumptions imply that we can use realized market portfolio returns to back out state prices and state probabilities. None of these assumptions are likely to hold absolutely, but they are quite similar to those made when implementing an unconditional CAPM model.[11]

Our definition of risk neutral alpha is tied to a particular interpretation of the term "alpha." In the parlance of conventional asset pricing, if the CAPM embodies all risk, then alpha is orthogonalized by construction and there is no sense in which it needs to be "risk adjusted." But most investors and economists control for market exposure in their assessment of performance not

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[11] The last assumption – that probabilities of good and bad states are constant – is probably the most problematic of these given the manner by which we construct our measure of bad times. Specifically, since the definition of bad times is based on the realization of the market portfolio in the previous period, the likelihood of ending up in a bad state is likely to vary over time. We note however that our drawdown measure of bad times is robust to this problem, as our drawdown measure uses the maximum drawdown to define the bad times. As we shall see later, our results are largely robust to using the drawdown definition of bad times.

because they believe this captures all risk in the portfolio, but because it is a step in the right direction.

Results: Risk neutral alphas of characteristics-sorted portfolios

The bottom rows of Table 3 show risk neutral alphas. Value, safety, total volatility, and momentum all have risk neutral alphas greater than 30 basis points per month. Characteristics measuring liquidity tend to have risk neutral alphas closer to zero.

For value, the risk neutral alpha is statistically significant and higher than the unconditional alpha. This results it consistent with Lakonishok, Shleifer, and Vishny (1994). Lakonishok, Shleifer, and Vishny who suggest that value stocks do not perform worse than growth stocks during bad times. While Lakonishok, Shleifer, and Vishny are concerned with the driver of the value premium and emphasize that value stocks do not underperform during bad times, we emphasize that value stocks actually achieve most of their risk neutral alpha during bad times.

Contrarily, Yogo (2006) finds that value stocks do indeed lose money in bad times as these stocks have high durable consumption betas. Since our bad times measure is correlated with durable consumption changes, our results seemingly differ from Yogo's. One reason for this apparent disconnect is a subtle bias that arises in Yogo's two-step GMM procedure. Namely, the correlation between consumption and stock returns is too low to reliably estimate consumption betas. Once this bias is corrected for, Bryzgalova (2015) shows that changes in durable consumption are not able to explain the value premium.[12]

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[12] Additionally, simply regressing changes in durable consumption on the returns to HML gives a negative, not positive, correlation, suggesting that, to a first order, HML is a hedge against consumption losses.

Illiquid stocks underperform during bad times and their risk neutral alphas are close to zero. These results are consistent with the pricing equation in (3) and with the theoretical results in Acharya and Pedersen (2005). Acharya and Pedersen's model suggest that a negative shock to liquidity causes contemporaneously lower returns to illiquid stocks. Indeed, as documented in Table 2, bad times are periods of high illiquidity, and the negative bad times alphas of the illiquidity factors are thus in line with Acharya and Pedersen's results.

The low risk factors have positive risk neutral alphas, but they have bad-times alphas close to zero. These results are potentially consistent with a leverage constraints based explanation for the low risk effect as suggested by Black (1972).[13] The momentum factor also has a positive risk adjusted alpha that is borderline significant. The high risk neutral alpha is consistent with behavioral theories of momentum (Barberis, Shleifer, Vishny, 1998, Daniel, Hirshleifer, and Subrahmanya, 1998, Hong and Stein, 1999), but the observation that momentum underperforms dramatically in bad times is not easily aligned with the behavioral explanations. On a practical level, momentum underperforms in bad times because 'momentum crashes' generally happen during bad times (Daniel and Moskowitzch, 2016). Under some of our alternate definitions, momentum does not crash during bad times and accordingly it has an even higher risk neutral alpha.

Much like CAPM alphas, our risk neutral alphas do not condition on the volatility of the strategy returns. Rescaling the reported alphas in Table 3 by the time series standard deviation of strategy returns (akin to an information ratio) yields nearly identical conclusions across the

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[13] Frazzini and Pedersen (2014) study the effect of leverage constraints in a dynamic model, and they find that tighter leverage constraints cause contemporaneously lower returns for low risk factors. To the extent that investors become more leverage constrained during bad times, our results are thus consistent with a leverage based explanation for the low risk effect.

characteristics. The two exceptions are for the profitability characteristic and the low volatility characteristics. Profitability has a low standard deviation, therefore strengthening our conclusion about its risk neutral performance; the low volatility portfolio has a high standard deviation because it is short high volatility stocks, and rescaling the low volatility portfolio to match the risk of the other strategies weakens our conclusion about its outperformance.

Relationship between Risk Neutral Alphas and CAPM Alphas

If there are only two states of a world, our risk neutral alpha is identical to the CAPM alpha. To see this, note that the CAPM implies the following stochastic discount factor

$$m_s^{\text{capm}} = \frac{1}{(1+r^f)} - \frac{E[r^{w,e}]}{(1+r^f)\operatorname{var}(r^{w,e})}\left(r_s^{w,e} - E[r^{w,e}]\right) \tag{9}$$

where m_s^{capm} is the value of the stochastic discount factor in state s and $r_s^{w,e}$ is the excess return to the total wealth portfolio in state s. The value of the stochastic discount factor in state g and b are then

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\begin{aligned}
m_b^{\text{capm}} &= \frac{1/(1+r^f)}{\pi_b - \dfrac{r_b^{w,e}}{r_g^{w,e}}\pi_b} \\[2em]
m_g^{\text{capm}} &= \frac{1/(1+r^f)}{\pi_g - \dfrac{r_g^{w,e}}{r_b^{w,e}}\pi_g}
\end{aligned}
\tag{10}
$$

If we assume that the total wealth portfolio is the market portfolio – as usually done in when implementing the CAPM – the two values of *m* are identical to the values used for the risk neutral alpha specified in eq (6).

The two approaches yield the same alpha because they both use the return to the market portfolio as the basis of the value of the stochastic discount factor. The difference between the two arises when there are more than two states, which is what the usual application of the CAPM implies. With more than two states, marginal utility for each state is not revealed by the returns to the market portfolio and the risk free rate, and we must resort to theory to figure out the value of the stochastic discount factor. The CAPM implies that the stochastic discount factor takes the form in (10), but this form is true only if the market portfolio is mean-variance efficient (Hansen and Richard, 1987) – and there is ample evidence that it is not.

Instead, our approach limits the world to two states determined on the basis of economic intuition. Our approach is bound to be incomplete, but the hope is that it is closer to reality than specifying the stochastic discount factor for a continuum of states based on the CAPM assumptions. The CAPM implicitly puts the same value on a loss in December 2008 and a loss in January 2004 (the market was up by 2.2 percent in both months), which is probably not something many investors would agree on.

Our method also relates to the conditional CAPM. In the conditional CAPM, betas may be time-varying. Lettau and Ludvigson (2001) and Lewellen and Nagel (2006) show that the value and the size factor both have higher betas when the expected return on the market portfolio is high. Gormsen and Jensen (2017) show that this result holds for most equity factors and that the result holds in other asset classes as well. This evidence of time-varying riskiness is, however, quite

different from the results in our paper: we focus on the conditional performance of the factors, not the conditional riskiness.[14]

Rainy Day Portfolios

We can draw on our results so far to design portfolios, encompassing potentially multiple characteristics, which outperform during bad times. The first, "Rainy Day Long Only", is a long only portfolio designed to pay a high excess return during bad times. The second, "Rainy Day Hedged", is a long/short portfolio that is long stocks that have high bad times alphas and short the market portfolio. The latter has the highest outperformance during bad times, but the investor must be able to hedge the market exposure to take full advantage of the outperformance.

We start with the Rainy Day Long Only portfolio. This portfolio is long stocks with the highest excess return during bad times, reflecting investors who cannot hedge their exposure to the market portfolio and care only about excess returns. Low volatility and high book-to-market are the two characteristics associated with the highest bad times excess return, and accordingly we base the long only portfolio on these. At the beginning of each month, the Rainy Day Long Only portfolio goes long stocks with book-to-market in the highest 30th percentile and total volatility in the lowest 30th percentile. The portfolio is value weighted, refreshed every month, and rebalanced every month.

The Rainy Day Long Only portfolio has an excess return of -0.75 percent per month during rainy days, which is substantially better than the -2.3 percent the market portfolio delivers. Since most stocks have positive betas, it is hard to construct a long-only portfolio that does not suffer

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[14] Gormsen and Jensen's results are nonetheless consistent with ours. The factors that have positive risk-neutral alphas also have positive conditional CAPM alphas, which is to say that neither their conditional performance nor their conditional riskiness can explain the factors' high average return.

during rainy days, but the Rainy Day Long Only portfolio diminishes the loss by two thirds. In addition to beating the market during bad times, the long only portfolio also beats the market during good times, delivering an average excess return during these months of 1.03 percent, compared to the market excess returns of 0.73 percent.

The Rainy Day Hedged portfolio is long characteristics that are associated with the highest bad times alpha. The four characteristics associated with the highest bad times alphas are small market cap, value, profitability, and a low bid-ask spread. We exclude the bid-ask spread, since it is only available from 1982, and we construct the rainy day portfolio as follows. At the beginning of each month, the long leg of the rainy day portfolio buys stocks that have all of the following: Book-to-market in the highest 30th percentile, profitability in the highest 30th percentile, and market value below the median. This procedure results in a portfolio of around thirty firms in the beginning of the sample and several hundred firms by the end of the sample. The long leg of the portfolio is value-weighted, refreshed every month, and rebalanced every month. The Rainy Day Hedged portfolio is long 1 dollar in the long leg and short 1.05 dollars in the market portfolio to hedge its beta exposure.

As shown in Table 4, the bad times alpha of the hedged portfolio is 2.29 percent per month (*t*-statistic 3.36). This bad times alpha is higher than any of the individual factors we consider. In addition, the rainy day portfolio also has a positive good times alpha of 0.68 percent per month, meaning that the high performance during bad times does not come at the cost of underperformance during good times. The rainy day portfolio has a risk neutral alpha of 1.2 percent per month, which is higher than any of the other factors we consider. Accordingly, investors with average risk preferences should prefer the rainy day portfolio over both the market and the other factors.

Figure 2 plots the cumulative log excess return to the Rainy Day Hedged portfolio and the market portfolio. The figure illustrates how the Rainy Day Hedged portfolio is fairly similar to the market during good times, but moves in the opposite direction to the market during bad times. The figure shows that the periods of underperformance tend to occur during the good times just before (but not during) financial crises.

Part of the reason why the Rainy Day Long Only portfolio has a high return during bad times, relative to the market, is that the Rainy Day Long Only portfolio has a low beta. Contrary to the Rainy Day Hedged portfolio, the Rainy Day Long Only portfolio does not have particularly high rainy day alphas or risk neutral alphas. Rather, the Rainy Day Long Only portfolio combines a high average alpha with a low beta to ensure downside protection at a low cost.

Finally, it is well known that combining profitability and value results in a high unconditional alpha (Novy-Marx, 2013). A natural concern, however, is that this unconditional alpha is compensation for poor performance during bad times. We show that not only is the value and profitability combination robust to controlling for bad times, it is actually the portfolio investors should hold if they are solely interested in generating high returns during bad times.

Hedging Against Bad Times Using Put Options

Another natural way to hedge against bad times is to buy long maturity put options. Since rainy days are periods where the market portfolio is in a drawdown, long maturity put options will generally increase in price during bad times, and thus serve as a hedge against these states. In this section, we show that investors can indeed hedge against bad times by using put options, but unlike the characteristics-based rainy day portfolios that we have described so far, using put options to

hedge against bad times is expensive during good times. Accordingly, the risk-neutral alpha of buying put options is close to zero.

We consider two simple put option strategies. Both strategies are long at-the-money S&P 500 put options with long maturities, but the maturity of the put options differs between the two strategies. The first strategy is long put options with maturity of one year. The second strategy is instead long the put option with the longest available maturity in the given month. At the end of each month, both strategies go long the appropriate put option, hold it for a month, and sell it again at the end of the subsequent month. We use end-of-day mid quotes as measures of price. All data is from the OptionMetrics database. The sample is from 1996 to 2013.

Table 5 shows the performance of the two strategies. As expected, both strategies have negative returns in good times and positive returns in bad times. Since both strategies have fairly low CAPM betas, the excess returns translates into a negative good times alpha and a positive bad times alpha. For the one-year maturity strategy, the difference between the good times and bad times alpha is statistically significant, as can be seen from the estimate on the bad times dunmy.

The good and bad times alphas for the two put option strategies appear consistent with the state prices we have identified using the market portfolio. Indeed, the risk neutral alpha for Strategy 2 is close to zero, and statistically insignificant. The near-zero risk-neutral alpha stands in stark contrast to the risk-neutral CAPM alpha shown at the bottom of the table. Consistent with much prior work, the CAPM alpha is large and negative, and it is statistically significant for the one-year maturity strategy.[15] In other words, the seemingly poor performance of the put-writing strategy is

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[15] See most recently Jurek and Stafford (2015)

best understood in a framework in which investors command a premium for assets that perform well in bad times. Our simple two state representation cleanly illustrates this logic.

IV.   Extensions

In this section, we describe three extensions. We first explore "defensive" industries, defined as industries that outperform during bad times. We then extend our results to international data, as well as repeating our main results using alternate definitions of bad times.

Defensive Industries

Table 6 shows the performance of the 49 Fama and French industry portfolios during good and bad times. The three industries with the worst absolute performance during bad times are real estate, construction, and machinery. The three industries with the best performance are Gold, smoking related, and coal. Looking across the 49 portfolios, there is only a modest correlation of 0.23 between rainy day alpha and unconditional CAPM alpha. This low correlation suggests that merely holding industries with high CAPM alpha will not result in a particularly high outperformance during rainy days. In addition, the correlation between bad times alpha and good times alpha is -0.15, which means that industries that tend to outperform in good times tend to underperform in bad times (and vice versa). For comparison, the risk factors studied earlier have a correlation between CAPM alpha and bad times alpha of -0.09 (0.23 for industries) and they have a correlation between good times alpha and bad times alpha of -0.37 (-0.15 for industries). These results suggest that the industries are less sensitive to the state of the economy than the risk factors are.

The level of the outperformance among the industries is modest: most industries have CAPM alphas and risk neutral alphas close to zero. Together the results suggest that protection

against bad times is more easily obtained through positions in the risk factors, or the Rainy Day portfolios, than through positions in the industry portfolios.

International Stocks during Bad Times

We repeat our main analysis from Table 3 using a broad sample of international stocks. We construct a set of international risk factors for our international robustness check. The international risk factors are the value-weighted return to twenty-four national risk factors. That is, the return to an international risk factor *f* is given by

$$r_{t+1}^{f,\text{global}} = \sum_{j=1}^{J} \frac{\mu_t^j}{\sum_j \mu_t^j} r_{t+1}^{f,j} \tag{11}$$

where μ_t^j is the market capitalization of country j at time t. The national risk factors are constructed similarly to the Fama and French portfolios, except that outside the U.S., the portfolio breakpoints are based on the largest 20 percent of the firms in a given country (rather than NYSE breakpoints).

Table 7 presents our main asset pricing test for the international sample. The results are similar to the results in the U.S. sample. For eight out of the nine factors we consider, the sign on the bad times dummy is the same in the international sample and in the U.S. sample. Additionally, size, value, and profitability have the largest bad times alphas both in the international sample and in the U.S. sample, which suggest a certain degree of robustness of the ranking of the factors' bad times alphas.

As in the U.S. sample, only value and profitability have statistically significant risk neutral alphas. The *t*-statistics of these two factors are approximately the same in the international sample and in the U.S. sample despite the international sample being substantially shorter. The high *t*-

statistics in the short international sample suggest that, for value and profitability, the ratio of risk-neutral alpha to volatility is larger in the international sample. For value, the higher ratio mainly comes from a lower volatility, whereas for profit, it mainly comes from a higher risk neutral alpha.

The only factor that differs substantially across the two samples is the growth factor. In the U.S., growth outperforms in good times but underperforms in bad times, whereas in the global sample, growth outperforms in bad times but underperforms in good times.

Robustness Tests on Definitions of Bad Times

In Table 1 we described alternate measures of bad times. Table 8 replicates our main asset pricing tests from Table 3 using these alternate measures. Recall that our main conclusion from Table 3 was that size, value, and profitability outperformed during bad times, and even more so than during good times. For size, this conclusion is robust across all our definitions of bad times, expect for financial crises. For value and profitability, this conclusion is robust across all our definitions of bad times. The value factor does, however, lose much of its bad times alpha when we consider the drawdown measure of bad times, reflecting that it does not do well in periods leading up to a rainy day period. Profitability, on the other hand, does better under our drawdown measure, reflecting that it does well in periods leading up to a rainy day period.

Our Rainy Day Hedged portfolio is also robust to different definitions of bad times. The Rainy Day Hedged portfolio struggles the most during the drawdown and the financial crisis definitions of bad times, but it still maintains a statistically significant monthly bad times alpha of 1.34 percent during these definitions.

Under our baseline definition of bad times, the good unconditional performance of illiquid stocks is explained by conditional underperformance in bad times. This result is intuitive, but it is

not robust to other definitions of bad times. During economic bad times, for instance, low turnover stocks and low Amihud ratio stocks do worse than in good times, but they still have a positive bad times alpha. During financial bad times, the turnover factor and the Amihud factor have negative bad times alphas, but the bad times alphas are not negative enough to justify the high good times alphas.

The safety, growth, and total volatility factors all have negative rainy day alphas, but this result is not robust to other definitions of bad times. While they have lower bad times alpha than good times alpha across all definitions of bad times, the good times alphas are sufficiently large for the bad times alphas to remain positive under most definitions of bad times.

V.   Conclusions

In this paper we analyze the cross-section of stock returns during bad times. We emphasize two contributions. First, we identify the characteristics of stocks that an investor who is worried about bad times should buy. For a long-only investor, this "rainy day" portfolio contains low volatility value stocks, and, counterintuitively, also performs reasonably well during good times. For an investor who can hedge her exposure to the overall stock market, the rainy day portfolio includes small, profitable value stocks.

Our second contribution is to propose a simple methodology to risk-adjust investment strategy returns in a state-contingent manner. The essence of the methodology is to place greater weight on performance achieved during bad times than performance achieved during good times, essentially evaluating returns under a risk neutral probability measure.

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Appendix: Risk Neutral Alphas

We describe here the general procedure for recovering risk neutral alphas for any strategy k, a set of defined states of nature S, and a set of reference factor portfolios used to estimate required returns in different states. The logic of our approach is similar to Hansen and Jagannathan (1991) who back out the restrictions on discount factors that would be required to price a given set of returns.

Consider an economy with S states of nature and $K \geq S$ factors. The pricing equation for factor k is:

$$1 = E[mR^k] = \sum_{s=1}^{S} m_s \pi_s R_s^k, \tag{A1}$$

where $R_s^k = 1 + r^f + r_s^k$ is 1 plus the risk free rate plus the excess return to factor k in state s, m_s is the stochastic discount factor in state s, and π_s is the probability of state s.

We can write (A1) for the risk free rate and the remaining $J = S-1$ factors in matrix form as:

$$1 = R\hat{\Pi} \tag{A2}$$

where 1 is an $S \times 1$ vectors of 1's, R is an $S \times S$ matrix where the first row is 1 plus the risk free rate, and the kth row contains the gross return to the k-1th factor in states 1 through S, and $\hat{\Pi}$ is an $S \times 1$ vector with $m_s\pi_s$ on the rows.

$$R = \begin{bmatrix} R^f & R^f & \cdots & R^f \\ R_1^1 & R_2^1 & \cdots & R_S^1 \\ \vdots & & & \vdots \\ R_1^J & \cdots & \cdots & R_S^J \end{bmatrix}, \text{ and} \tag{A3}$$

$$\hat{\Pi} = \begin{bmatrix} m_1 \pi_1 \\ \vdots \\ m_S \pi_S \end{bmatrix}. \tag{A4}$$

If none of the J assets are linear combinations of the other along the S-1 states, then $\boldsymbol{R}$ has full rank and we can solve for and $\hat{\Pi}$ as

$$\hat{\Pi} = \boldsymbol{R}^{-1} 1 \tag{A5}$$

We define the risk-neutral alpha for any strategy as

$$\alpha^{k,\mathrm{m}} \equiv \frac{E[mr^{k,\mathrm{e}}]}{E[m]} = (1+r^f)\hat{\Pi}r^k \tag{A6}$$

where $\boldsymbol{r^k}$ is an $S \times 1$ vector of excess returns in the S states for factor k. The above is the expected return under the risk-neutral measure.

Figure 1: Measuring bad times

This figure shows the time-series of bad times, marked by grey bars. Bad times are defined as quarters with both economic and financial bad times. Financial bad times are quarters where either the quarterly or the 12-month market excess returns is in the lowest quintile of the time-series. Economic bad times are quarters for which at least one of the months is defined as a recession by NBER. In the international sample, Economic bad times are quarters where the growth in the OECD countries' aggregate GDP is in the lowest quintile, and Financial bad times are quarters where either the quarterly or the yearly excess return to the global market portfolio is in the lowest quintile of the time-series.

Panel A: US time-series 1963-2013.
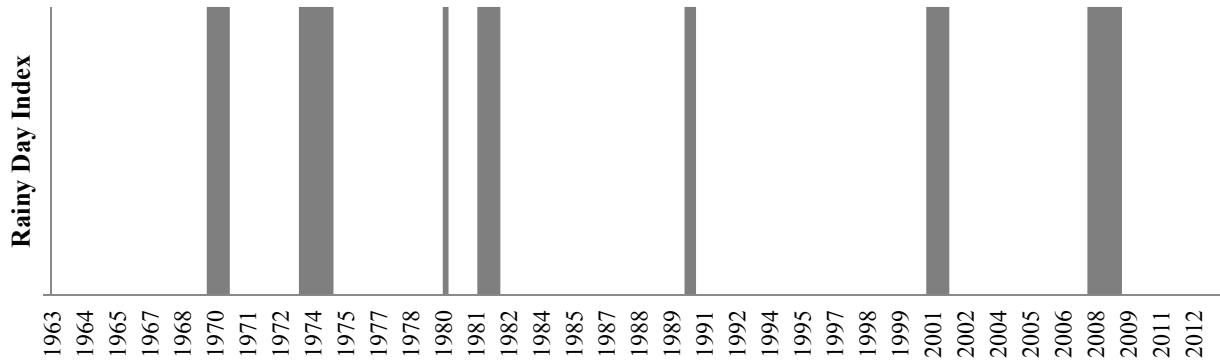

Panel B: International Bad Times, 1987-2013
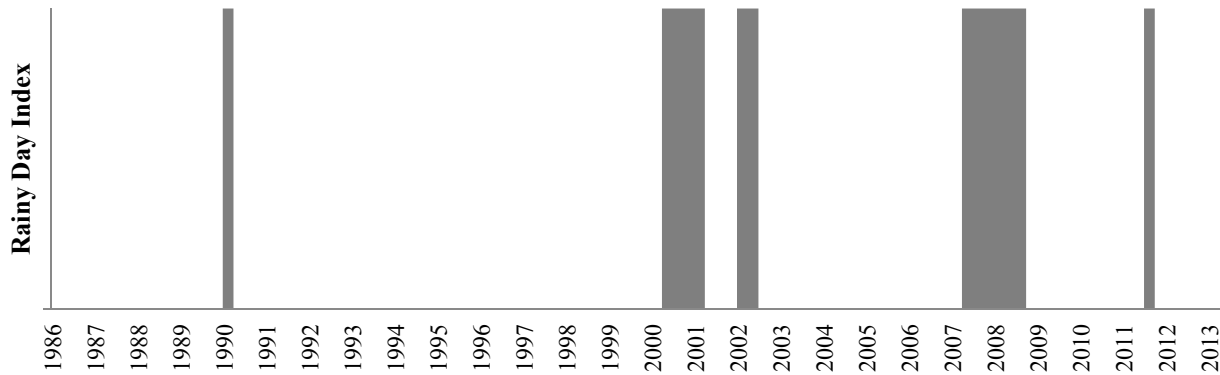
Figure 2. Cumulative Return to Rainy Day portfolios

This figure shows the cumulative log excess return to the rainy day portfolios and the market portfolio. Panel A shows the return to the Rainy Day Hedged portfolio and Panel B shows the return to the Rainy Day Long Only portfolio. Rainy days are marked with gray bars. Rainy days are defined as quarters with both economic and financial bad times. Financial bad times are quarters where either the quarterly or the yearly market excess returns is in the lowest quintile. Economic bad times are quarters for which at least one month is defined as a recession by the NBER. The Rainy Day portfolios are constructed as follows. The long leg of the Rainy Day Hedged portfolio consists of stocks that have all of the following: Book-to-market in the highest 30th percentile, profitability in the highest 30th percentile, and market value below the median. The long leg is value-weighted, refreshed every month, and rebalanced every month. The Rainy Day Hedged portfolio is long one dollar in the long leg and short 1.05 dollars in the market portfolio. The Rainy Day Long Only portfolio consists of stocks that have all of the following: Book-to-market in the highest 30th percentile and volatility in the lowest 30th percentile. The long leg is value-weighted, refreshed every month, and rebalanced every month. The sample is 1963-2013.

Panel A. Cumulative return to the Rainy Day Hedged portfolio
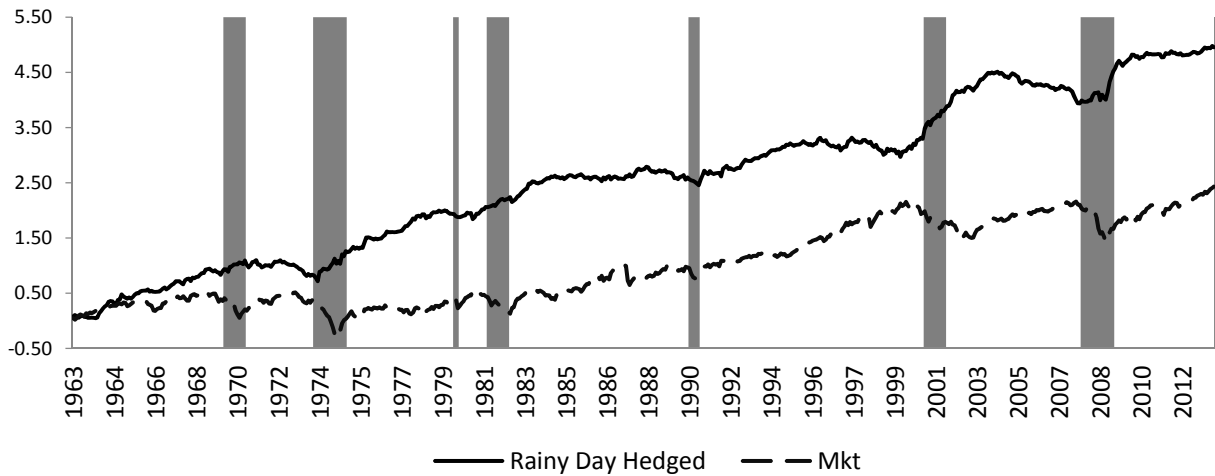
Panel B. Cumulative return to the Rainy Day Long Only portfolio
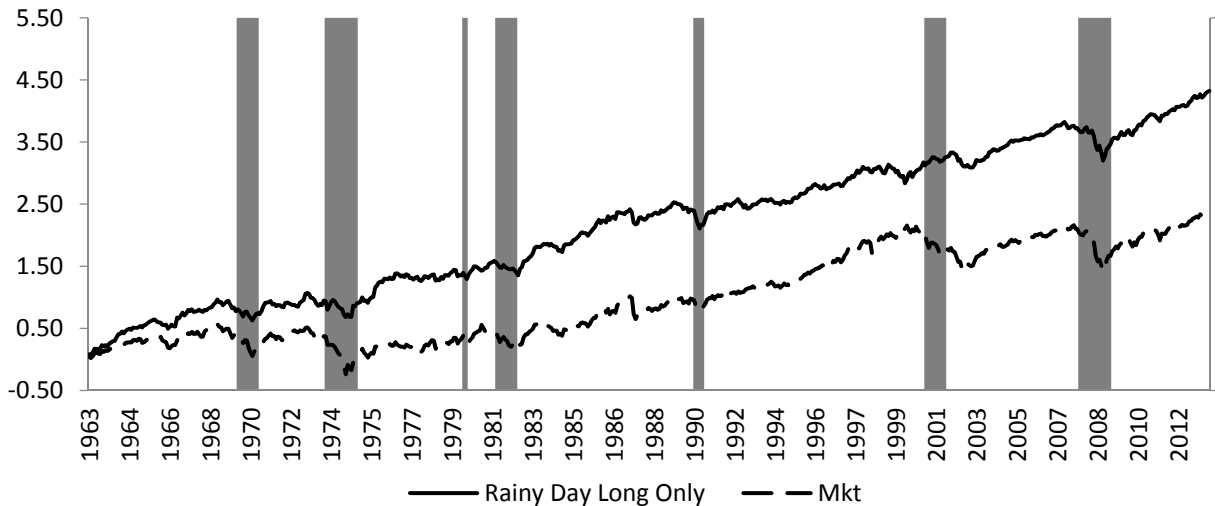
Table 1. Bad Times

This table reports time-series averages of economic and financial variables during bad times. Bad times are defined as quarters with both economic and financial bad times. Financial bad times are quarters where either the quarterly or the yearly market excess returns is in the lowest quintile of the time-series. Economic bad times are quarters for which at least one of the months is defined as a recession by NBER. Drawdown is quarters that are between the beginning and the maximum of a drawdown that reaches at least 10 percent. The market excess return is the excess return to the CRSP value-weighted portfolio. GDP growth is the quarterly change in real GDP. Inflation is the quarterly change in the consumer price index. Earnings growth is the quarterly change in one-year earnings of the stock market. The change in the home price index is the quarterly change in the Case-Shiller home price index. Unemployment is measured at the end of each quarter. The Noise measure is the noise in the term structure as defined by Hu, Pan, Wang (2013). The Noise measure, the TED spread, and the VIX are measured at the end of each quarter. All numbers are expressed in percent except the noise measure and the change in the home price index. All percentages are annualized. The sample is from 1963-2013 in the U.S.

			Alternate Ways to Identify Bad Times							
		Baseline: All Quarters	Bad times: Baseline	Finance	Econo mic	Finance only	Econ only	Draw-down	All	Global
1963-2013	Quarters	204	27	50	34	23	7	47	22	14
Market returns (%)										
1963-2013	Average excess market	6.3	-19.4	-22.1	-7.4	-25.2	39.0	-30.2	-36.7	-15.7
1963-2013	Volatility of market	17.2	26.0	22.5	26.7	17.8	13.6	16.0	19.4	26.0
Economic growth and inflation (%)										
1963-2013	Average GDP growth	3.0	-1.1	1.1	-1.3	3.6	-2.0	1.6	-1.0	
	Global GDP growth									-0.9
1963-2013	Average earnings growth	4.0	-24.3	-6.2	-21.9	14.9	-13.0	-12.1	-22.0	
	Volatility of earnings	18.5	18.7	22.3	16.8	22.0	2.7	13.9	16.2	
1963-2013	Average inflation	4.0	5.9	5.1	5.7	4.2	4.9	5.2	5.9	
	Volatility of inflation	1.9	3.1	2.5	2.9	1.4	2.2	2.6	3.5	
1963-2013	Average unemployment	6.1	6.3	6.1	6.6	5.8	7.7	5.7	6.1	
1963-2013	Δ in home price index	0.7	-5.2	-1.4	-5.4	3.1	-6.2	-2.7	-6.5	
Liquidity measures (%)										
1985-2013	Average TED spread	0.6	0.9	0.7	1.0	0.4	1.4	0.9	1.0	
1985-2013	Average VIX	21.0	30.9	31.8	29.6	32.6	21.2	30.2	32.3	
1987-2013	Term structure noise	3.5	6.7	5.2	6.4	3.7	4.4	5.5	7.1	

Table 2. Characteristics of Stocks with Ex Post Good Performance during Bad Times

This table shows characteristics of portfolios sorted on ex-post performance of US stocks during bad times occurring between 1963 and 2013. All characteristics are measured as percentile ranks. For example, column 2 of Panel A shows that the worst performing stocks during bad times disproportionately contain growth stocks. Panel A shows portfolios sorted on excess returns, and Panel B shows portfolios sorted on CAPM alpha. We pool series of consecutive Rainy Day quarters into a total of seven periods and calculate returns and CAPM alpha for each stock each period. For each period, we form a set of portfolios based on quintiles of excess return or CAPM alpha. For each period and portfolio, we estimate ex ante characteristics of the portfolios based on equal weights. The final characteristic estimate for each portfolio is then the weighted time series average across the seven periods of consecutive rainy days. The time series weights are the number of quarters each period lasts. Characteristics are measured in percentiles of the cross-section. CAPM alpha is the excess return on the stock minus the product of the excess market return and the ex ante estimate of the stock's beta. Rainy days are defined as quarters with both economic and financial bad times. More details on the construction of characteristics (size, book-to-market, and so on) is provided in the main text.

Panel A: Characteristics of Stocks Sorted on Ex Post Excess Returns during Bad Times (all measured as percentile ranks)

		Size and Value		Good Fundamentals		Low Risk		Momentum	Illiquidity		
		Size	BM	Profit	Growth	Safety	Total Volatility	Momentum	Amihud Ratio	Turnover	Bid-Ask Spread
(low return)	1	43.5	48.5	43.6	51.0	37.4	66.2	44.5	47.8	59.0	44.3
	2	51.4	47.7	50.6	51.7	49.8	51.1	50.3	49.2	51.4	52.4
	3	55.7	49.7	52.2	50.9	55.0	42.6	53.8	48.7	47.2	55.4
	4	55.4	49.4	55.0	48.8	58.8	38.7	54.6	50.3	44.6	52.6
(high return)	5	48.2	53.6	50.7	47.6	52.5	46.5	49.5	53.1	46.3	49.8
5 - 1		**4.7**	**5.1**	**7.1**	**-3.4**	**15.0**	**-19.7**	**5.1**	**5.3**	**-12.7**	**5.5**

Panel B: Characteristics of Stocks Sorted on Ex Post Alpha During Bad Times (all measured as percentile ranks)

		Size and Value		Good Fundamentals		Low Risk		Momentum	Illiquidity		
		Size	BM	Profit	Growth	Safety	Total Volatility	Momentum	Amihud Ratio	Turnover	Bid-Ask Spread
(low alpha)	1	43.2	48.9	45.4	49.8	42.8	59.1	47.2	51.0	52.8	47.7
	2	51.7	49.5	50.4	50.0	53.7	44.7	52.2	50.5	47.3	55.5
	3	55.8	48.8	53.3	50.4	56.7	41.8	53.8	48.7	47.0	55.0
	4	56.4	48.6	53.7	50.7	54.9	43.4	52.5	48.1	48.5	50.3
(high alpha)	5	48.3	52.4	49.9	49.8	45.7	55.9	47.5	49.6	54.1	44.7
5 - 1		**5.1**	**3.4**	**4.5**	**0.0**	**2.9**	**-3.2**	**0.3**	**-1.4**	**1.3**	**-3.0**

Table 3. Performance of Characteristic-sorted Portfolios during Bad Times

This table reports the conditional performance of characteristic-sorted portfolios based on US stocks 1963-2013. Panel A shows summary statistics of each risk factor during rainy day months and good months. Panel B shows conditional alphas for each risk factor. For each risk factor, we run a regression of monthly excess return on a dummy variable and the monthly excess market return:

$$r_t^{p,e} = \alpha_u + \alpha_b 1_b + \beta(r_t^m - r^f) + u_t$$

where 1_b is a dummy equal to 1 if the month is defined as a rainy day month and zero otherwise. Risk neutral alpha is the weighted sum of the average good times return and the average rainy day return where the weights are the risk neutral probabilities as implied by the market portfolio and the risk-free rate.

$$\alpha^{p,m} \equiv \frac{E[mr^{p,e}]}{E[m]}$$

The risk neutral probabilities are 0.33 for rainy days and 0.67 for good times. Rainy days are defined as quarters with both economic and financial bad times. Financial bad times are quarters where either the quarterly or the yearly market excess returns is in the lowest quintile. Economic bad times are quarters for which at least one month is defined as a recession by the NBER. *t*-statistics are reported below parameter estimates in parenthesis and statistical significance at the five percent level is indicated in bold. Returns are alphas are in monthly percent. Sharpe Ratios and Information ratios are annualized. More details on the construction of characteristics (size, book-to-market, and so on) are provided in the main text.

Panel A. Summary Statistics of Long-Short Portfolios

Characteristic:	Size and Value		Good Fundamentals		Low Risk		Momentum	Illiquidity		
	Size	Value-Growth	Profit	Growth	Safety	Total Volatility	UMD	Amihud	Turnover	Bid-ask
Long side of portfolio:	Small	Value	Profitable	Growing	Safe	Low risk	Winners	Illiquid	Low turnover	High Bid- Ask
Number of months	612	612	612	612	612	612	612	612	612	384
Number of rainy months	81	81	81	81	81	81	81	81	81	42
Excess return good times (%)	**0.34**	**0.27**	**0.17**	**0.22**	0.04	-0.02	**0.76**	-0.06	-0.10	0.10
	(2.71)	(2.14)	(2.01)	(3.21)	(0.23)	(-0.09)	(5.25)	(-0.78)	(-0.63)	(0.70)
Excess return rainy days (%)	0.12	1.19	**0.62**	-0.01	1.07	1.41	0.06	0.31	0.82	0.07
	(0.30)	(1.92)	(2.38)	(-0.05)	(1.54)	(1.68)	(0.08)	(1.08)	(1.35)	(0.08)
Sharpe Ratio (good times)	0.41	0.32	0.30	0.48	0.03	-0.01	0.79	-0.12	-0.09	0.13
Sharpe Ratio (bad times)	0.12	0.74	0.92	-0.02	0.59	0.65	0.03	0.41	0.52	0.04
Inf. Ratio (good times)	0.22	0.38	0.48	0.60	0.76	0.63	0.90	0.22	0.48	0.65
Inf. Ratio (bad times)	0.45	0.67	0.67	-0.20	-0.16	-0.01	-0.09	-0.06	-0.21	-0.42

Panel B. Rainy Day Alphas and Risk Neutral Alphas

Characteristic:	Size and Value		Good Fundamentals		Low risk		Momentum	Liquidity factors		
	Size	Value-Growth	Profit	Growth	Safety	Total Volatility	UMD	Amihud	Turnover	Bid-ask
Long side of portfolio:	Small	Value	Profitable	Growing	Safe	Low vol	Winners	Illiquid	Low turnover	High Bid- Ask
α_u: Good times alpha	0.18	**0.32**	**0.25**	**0.27**	**0.65**	**0.67**	**0.89**	0.11	**0.40**	**0.43**
(t)	(1.42)	(2.14)	(3.04)	(3.80)	(4.77)	(3.97)	(4.93)	(1.34)	(2.99)	(2.93)
α_b: Rainy day dummy	0.27	0.77	0.18	-0.37	**-0.84**	-0.68	**-1.08**	-0.15	-0.63	**-0.99**
(t)	(0.76)	(1.87)	(0.78)	(-1.94)	(-2.21)	(-1.46)	(-2.17)	(-0.67)	(-1.68)	(-2.25)
β	**0.19**	-0.06	**-0.11**	**-0.06**	**-0.73**	**-0.82**	**-0.15**	**-0.20**	**-0.60**	**-0.34**
(t)	(7.08)	(-1.85)	(-6.08)	(-3.90)	(-25.49)	(-23.27)	(-3.92)	(-12.28)	(-21.41)	(-11.24)
$\alpha_u + \alpha_b$: Rainy day alpha	0.45	**1.09**	**0.43**	-0.11	-0.18	-0.01	-0.19	-0.04	-0.22	-0.56
(t)	(1.38)	(2.87)	(2.03)	(-0.60)	(-0.53)	(-0.02)	(-0.42)	(-0.20)	(-0.65)	(-1.37)
Risk neutral alpha	0.27	**0.57**	**0.31**	0.14	0.38	0.45	0.53	0.06	0.20	0.09
(t)	(1.69)	(2.58)	(3.05)	(1.77)	(1.46)	(1.44)	(1.86)	(0.53)	(0.87)	(0.35)

Table 4. Rainy Day Portfolios

This table describes the performance of portfolios constructed to maximize performance during bad times. We consider two portfolios: a market neutral "rainy day hedged" portfolio, and a "rainy day long only" portfolio. Portfolios are constructed as follows. The long leg of the Rainy Day Hedged portfolio consists of stocks that have all of the following: Book-to-market in the highest 30th percentile, profitability in the highest 30th percentile, and market value below the median. The long leg is value-weighted, refreshed every month, and rebalanced every month. The Rainy Day Hedged portfolio is long one dollar in the long leg and short 1.05 dollars in the market portfolio. The Rainy Day Long Only portfolio consists of stocks that have all of the following: Book-to-market in the highest 30th percentile and volatility in the lowest 30th percentile. The long leg is value-weighted, refreshed every month, and rebalanced every month. t-statistics are reported below parameter estimates in parenthesis and statistical significance at the five percent level is indicated in bold. After summarizing excess returns, Sharpe ratios, and information ratios during good and bad times, we the report the results of a time series regression of monthly excess return on a dummy variable and the monthly excess market return:

$$r_t^{p,e} = \alpha_u + \alpha_b 1_b + \beta(r_t^m - r^f) + u_t$$

where 1_b is a dummy equal to 1 if the month is defined as a rainy day month and zero otherwise. Risk neutral alpha is the weighted sum of the average good times return and the average rainy day return where the weights are the risk neutral probabilities as implied by the market portfolio and the risk-free rate.

$$\alpha^{p,m} \equiv \frac{E[mr^{p,e}]}{E[m]}$$

The risk neutral probabilities are 0.33 for rainy days and 0.67 for good times. The risk neutral probabilities are 0.33 for rainy days and 0.67 for good times. Rainy days are defined as quarters with both economic and financial bad times. The sample is 1963-2013.

	Rainy Day Hedged Portfolio	Rainy Day Long Only Portfolio
Number of months	612	612
Number of rainy months	81	81
Excess return good times	**0.68**	**1.03**
	(4.42)	(6.38)
Excess return during bad times	**2.29**	-0.75
	(3.36)	(-1.18)
SR good times	0.66	0.96
SR bad times	1.29	-0.45
IR good times	0.66	0.61
IR bad times	1.29	0.40
α_u: Good times alpha	**0.68**	**0.47**
(t)	(3.86)	(3.83)
Rainy day dummy	**1.61**	-0.08
	(3.34)	(-0.23)
Beta	0.00	**0.66**
	(0.03)	(25.71)
Risk neutral alpha	**1.20**	0.44
	(4.73)	(1.89)

Table 5. Good and Bad Times Performance of Buying At the Money Put Options

This table describes the performance of portfolios that is long deep out of the money put options. We consider two portfolios: the first portfolio is long put options with one-year maturity and the second portfolio is long the put option with longest available maturity on the given day. The portfolios are constructed at the end of each month and held for one month. After summarizing excess returns, Sharpe ratios, and information ratios during good and bad times, we report the results of a time series regression of monthly excess return on a dummy variable and the monthly excess market return:

$$r_t^{p,e} = \alpha_u + \alpha_b 1_b + \beta(r_t^m - r^f) + u_t$$

where 1_{bt} is a dummy equal to 1 if the month is defined as a rainy day month and zero otherwise. Risk neutral alpha is the weighted sum of the average good times return and the average rainy day return where the weights are the risk neutral probabilities as implied by the market portfolio and the risk-free rate.

$$\alpha^{p,m} \equiv \frac{E[mr^{p,e}]}{E[m]}$$

The risk neutral probabilities are 0.33 for rainy days and 0.67 for good times. The risk neutral probabilities are 0.33 for rainy days and 0.67 for good times. Rainy days are defined as quarters with both economic and financial bad times. The sample is 1963-2013.

	Portfolio 1: Buy 1-yr put options on S&P 500	Portfolio 2: Buy longest maturity available options
Number of months	213	213
Number of rainy months	30	30
Excess return good times	**-2.34**	**-1.45**
	-(3.44)	-(2.11)
Excess return during rainy days	2.37	2.51
	(0.98)	(1.05)
SR good times	-0.88	-0.54
SR bad times	0.62	0.66
IR good times	-0.84	-0.48
IR bad times	0.57	0.61
α_u: Good times alpha	**-2.24**	-1.29
(t)	-(3.01)	-(1.74)
Rainy day dummy	**4.41**	3.54
	(2.22)	(1.78)
Beta	-0.12	-0.16
	-(0.80)	-(1.08)
Risk neutral alpha	-0.73	-0.09
	-(0.76)	-(0.10)
CAPM alpha	**-1.58**	-0.77
	-(2.30)	-(1.12)

Table 6. Defensive Industries

This table reports conditional performance of forty-nine Fama and French industries. Industries are sorted by their average monthly performance during bad times. For each portfolio, we run a regression of monthly excess return on a dummy variable and the monthly excess market return:

$$r_t^{p,e} = \alpha_u + \alpha_b 1_b + \beta(r_t^m - r^f) + u_t$$

where 1_b is a dummy equal to 1 if the month is defined as a rainy day month and zero otherwise. Good times alpha is the intercept α_u. Rainy days alpha is the intercept α_u plus the parameter estimate of the dummy variable α_{bt}. The CAPM alpha is the intercept in a time-series regression of excess return on the market excess return. Risk neutral alpha is the weighted sum of the average good times return and the average rainy day return where the weights are the risk neutral probabilities as implied by the market portfolio and the risk-free rate.

$$\alpha^{p,\mathrm{rn}} \equiv \frac{E[mr^{p,e}]}{E[m]}$$

The risk neutral probabilities are 0.33 for rainy days and 0.67 for good times. Rainy days are defined as quarters with both economic and financial bad times. Financial bad times are quarters where either the quarterly or the yearly market excess returns is in the lowest quintile. Economic bad times are quarters for which at least one month is defined as a recession by the NBER. *t*-statistics are reported below parameter estimates in parenthesis and statistical significance at the five percent level is indicated in bold. Returns and alphas are in monthly percent. The sample is from 1963-2013.

#	Industry	Excess returns (% monthly)			Alphas (% monthly)			
		Average	Good times	Bad Times	Good times (α_u)	Bad Times ($\alpha_u + \alpha_b$)	CAPM	Risk neutral
47	Real Estate	0.33	0.86	-3.16	-0.10	-1.19	-0.25	-0.46
18	Construction	0.56	1.02	-2.40	-0.08	-0.15	-0.09	-0.10
21	Mach	0.55	0.99	-2.35	-0.04	-0.25	-0.07	-0.11
45	Banks	0.45	0.86	-2.25	0.00	-0.48	-0.07	-0.16
24	Aero	0.77	1.23	-2.24	0.28	-0.31	0.20	0.09
38	Lab Equipment	0.72	1.17	-2.21	0.01	0.18	0.03	0.06
6	Toys	0.56	0.98	-2.20	-0.15	0.12	-0.11	-0.06
8	Books	0.64	1.07	-2.18	0.19	-0.37	0.11	0.01
37	Chips	0.57	0.99	-2.18	-0.16	0.18	-0.11	-0.05
46	Insurance	0.55	0.96	-2.15	0.21	-0.61	0.10	-0.06
22	Electical Eq.	0.71	1.13	-2.09	0.14	-0.06	0.11	0.08
48	Financial Services	0.64	1.06	-2.08	0.12	-0.15	0.08	0.03
34	Business Services	0.58	0.97	-1.99	-0.05	0.10	-0.03	0.00
44	Meals	0.74	1.15	-1.97	0.24	-0.10	0.19	0.13
11	Health	0.67	1.13	-1.91	0.18	0.02	0.15	0.13
23	Autos	0.47	0.84	-1.91	-0.12	0.07	-0.10	-0.06
36	Software	0.52	0.90	-1.88	-0.40	0.86	-0.22	0.01
17	Building Materials	0.64	1.02	-1.84	0.05	0.15	0.07	-0.03
20	FabPr	0.50	0.85	-1.84	-0.10	0.11	-0.07	0.08
15	Rubber	0.54	0.89	-1.80	-0.07	0.17	-0.03	0.01
35	Hardware	0.57	0.92	-1.78	-0.03	0.18	0.00	0.04
28	Mines	0.62	0.98	-1.75	0.01	0.24	0.05	0.09
41	Transportation	0.55	0.88	-1.65	-0.03	0.22	0.00	0.05
9	Household	0.57	0.90	-1.63	0.18	-0.14	0.13	0.07
14	Chemicals	0.56	0.89	-1.60	0.04	0.15	0.05	0.07
49	Other	0.47	0.78	-1.58	-0.03	0.08	-0.01	0.01
1	Agricultural	0.46	0.76	-1.57	-0.01	0.02	-0.01	0.16
30	Oil	0.66	1.00	-1.57	0.34	-0.21	0.26	0.00

		Excess returns (% monthly)			Alphas (% monthly)			
#	Industry	Average	Good times	Bad Times	Good times (α_u)	Bad Times ($\alpha_u + \alpha_b$)	CAPM	Risk neutral
10	Clothing	0.58	0.90	-1.52	0.00	0.34	0.04	0.11
25	Ships	0.75	1.09	-1.47	0.24	0.27	0.25	0.25
19	Steel	0.43	0.71	-1.46	-0.31	0.65	-0.18	0.00
16	Textiles	0.52	0.81	-1.39	-0.14	0.55	-0.04	0.09
7	Fun	0.86	1.20	-1.37	0.12	0.86	0.22	0.36
40	Boxes	0.62	0.92	-1.31	0.14	0.27	0.16	0.18
33	Personal Services	0.38	0.63	-1.29	-0.36	0.74	-0.21	0.00
32	Telecom	0.47	0.73	-1.24	0.07	0.11	0.08	0.08
3	Soda	0.79	1.09	-1.20	0.48	0.05	0.42	0.34
31	Utilities	0.41	0.65	-1.19	0.20	-0.25	0.13	0.27
42	Wholesale	0.69	0.98	-1.19	0.09	0.63	0.17	0.05
39	Paper	0.57	0.84	-1.15	-0.02	0.60	0.07	0.19
12	Med. Equipment	0.76	1.04	-1.10	0.30	0.41	0.32	0.34
43	Retail	0.61	0.86	-1.02	0.04	0.66	0.12	0.24
13	Drugs	0.68	0.94	-0.99	0.27	0.38	0.28	0.30
26	Guns	0.84	1.10	-0.86	0.35	0.68	0.39	0.46
2	Food	0.64	0.82	-0.51	0.26	0.65	0.31	0.39
4	Beer	0.72	0.89	-0.44	0.27	0.85	0.35	0.46
27	Gold	0.55	0.70	-0.41	0.03	0.96	0.16	0.34
5	Smoke	1.04	1.22	-0.13	0.63	1.09	0.69	0.78
29	Coal	0.56	0.66	-0.05	-0.34	1.99	-0.01	0.43

Table 7. International Results

This table reports conditional performance of characteristic-sorted portfolio in the international sample 1987-2013. For each risk factor, we run a regression of monthly excess return on a dummy variable and the monthly excess return to the global market portfolio:

$$r_t^{p,\text{e}} = \alpha_u + \alpha_b 1_b + \beta(r_t^m - r^f) + u_t$$

where 1_b is a dummy equal to 1 if the month is defined as a bad times month and zero otherwise. Risk neutral alpha is the weighted sum of the average good times return and the average return during bad times where the weights are the risk neutral probabilities as implied by the market portfolio and the risk-free rate. The risk neutral probabilities are 0.65 for rainy days and 0.35 for good times. The market excess return is the excess return to the value weighted portfolio of all firms. The national portfolios are constructed as the intersection of six value-weighted portfolios formed on size and the relevant characteristic. The size break-point is the median NYSE market equity in the U.S. and the 80th percentile of market equity in international countries. The Total volatility, Amihud, Turnover, or Bid-ask spread breakpoints are the 30th and 70th percentile conditional on size. Portfolios are refreshed every calendar month and rebalanced every calendar month to maintain value weights. *t*-statistics are reported below parameter estimates in parenthesis and statistical significance at the five percent level is indicated in bold. Alphas are in monthly percent.

	Size and Value		Good Fundamentals		Low risk		Momentum	Liquidity factors	
Characteristic:	Size	Value-Growth	Profit	Growth	Safety	Total Volatility	UMD	Amihud	Turnover
Long side of portfolio:	Small	Value	Profitable	Growing	Safe	Low vol	Winners	Illiquid	Low turnover
Number of months	327	327	327	327	327	327	327	327	327
Number of rainy months	42	42	42	42	42	42	42	42	42
α_u: Good times alpha	0.06	0.28	**0.42**	-0.04	**0.24**	**0.70**	**0.92**	0.07	0.26
(t)	(0.50)	(1.62)	(4.99)	(-0.39)	(2.29)	(3.52)	(4.12)	(0.76)	(1.69)
α_b: Rainy day dummy	0.39	**0.99**	0.19	0.37	-0.09	-0.63	**-1.34**	-0.53	-0.66
(t)	(1.10)	(2.09)	(0.82)	(1.44)	(-0.31)	(-1.13)	(-2.16)	-(1.94)	(-1.56)
β	**0.06**	0.05	**-0.19**	-0.01	**-0.28**	**-0.65**	**-0.24**	**-0.24**	**-0.48**
(t)	(2.44)	(1.46)	(-11.08)	(-0.40)	-(13.48)	(-16.25)	(-5.48)	-(12.50)	(-15.61)
$\alpha_u + \alpha_b$: Rainy day alpha	0.45	**1.27**	**0.62**	0.33	0.15	0.08	-0.42	-0.45	-0.41
(t)	(1.37)	(2.88)	(2.81)	(1.40)	(0.56)	(0.15)	(-0.74)	(-1.80)	(-1.03)
Risk neutral alpha	0.20	**0.62**	**0.49**	0.09	0.20	0.48	0.44	-0.11	0.02
(t)	(1.46)	(2.28)	(3.49)	(0.86)	(1.05)	(1.10)	(1.03)	(-0.79)	(0.09)
Rainy Day dummy same sign as US?	YES	YES	YES	NO	YES	YES	YES	YES	YES

Table 8. Robustness to Definitions of Bad Times

This table repeats the analysis from Panel B of Table 3 under alternate definitions of bad times. Rainy days are defined as quarters with both economic and financial bad times. Financial bad times are quarters where either the quarterly or the yearly market excess returns is in the lowest quintile. Economic bad times are quarters for which at least one month is defined as a recession by the NBER. Drawdown is quarters between the beginning and the maximum of a drawdown that reaches at least 10 percent. All is the intersection of all our measures of bad times. *t*-statistics are reported below parameter estimates in parenthesis and statistical significance at the five percent level is indicated in bold. Alphas are in monthly percent. The sample is 1963-2013.

	Size and value		Good Fundamentals		Low Risk		Momentum	Liquidity factors			Com-posite
	SMB	Value-Growth	Profit	Growth	Safety	Total Volatility	UMD	Amihud	Turnover	Bid-Ask	Rainy day hedged
Number of observations:	612	612	612	612	612	612	612	612	612	384	612
Rainy days (Baseline from Table 3):											
α_u: Good times alpha	0.18	0.32	0.25	0.27	0.65	0.71	0.89	0.11	0.40	0.43	0.67
(t)	(1.42)	(2.14)	(3.04)	(3.80)	(4.77)	(4.44)	(4.93)	(1.34)	(2.99)	(2.93)	(3.83)
α_b: Bad times dummy	0.27	0.77	0.18	-0.37	-0.84	-0.77	-1.08	-0.15	-0.63	-0.99	1.62
(t)	(0.76)	(1.87)	(0.78)	(-1.94)	(-2.21)	(-1.75)	(-2.17)	(-0.67)	(-1.68)	(-2.25)	(3.34)
$\alpha_u + \alpha_b$: Alpha during rainy days	0.45	**1.09**	**0.43**	-0.11	-0.18	-0.06	-0.19	-0.04	-0.22	-0.56	**2.29**
(t)	(1.38)	(2.87)	(2.03)	(-0.60)	-(0.53)	-(0.16)	-(0.42)	(-0.20)	(-0.65)	(-1.37)	(5.14)
Financial bad times:											
α_u: Good times alpha	0.26	0.28	**0.20**	**0.22**	**0.66**	**0.69**	**1.01**	0.15	**0.43**	**0.41**	**0.73**
(t)	(1.85)	(1.70)	(2.19)	(2.87)	(4.41)	(3.98)	(5.17)	(1.70)	(2.93)	(2.56)	(3.78)
α_b: Bad times dummy	-0.15	0.58	0.31	-0.02	-0.48	-0.36	**-1.06**	-0.24	-0.44	-0.41	0.65
(t)	(-0.52)	(1.74)	(1.64)	(-0.14)	-(1.55)	-(1.01)	(-2.64)	(-1.32)	(-1.46)	(-1.20)	(1.65)
$\alpha_u + \alpha_b$: Alpha during bad economic times	0.11	**0.86**	**0.51**	0.20	0.18	0.33	-0.05	-0.09	-0.01	0.00	**1.38**
	(0.45)	(3.03)	(3.19)	(1.48)	(0.71)	(1.10)	(-0.14)	(-0.58)	(-0.05)	(0.00)	(4.11)

[Continued overleaf]

	Size and value		Good Fundamentals		Low Risk		Momentum	Liquidity factors			Composite
	SMB	Value-Growth	Profit	Growth	Safety	Total Volatility	UMD	Amihud	Turnover	Bid-Ask	Rainy day hedged
Economic bad times:											
α_u: Good times alpha	0.18	**0.36**	**0.21**	**0.24**	**0.62**	**0.65**	**0.83**	0.10	**0.37**	**0.42**	**0.67**
(t)	(1.36)	(2.36)	(2.50)	(3.44)	(4.44)	(4.02)	(4.54)	(1.21)	(2.70)	(2.83)	(3.74)
α_b: Bad times dummy	0.24	0.40	0.39	-0.18	-0.49	-0.31	-0.56	-0.07	-0.32	-0.73	**1.33**
(t)	(0.77)	(1.08)	(1.89)	(-1.05)	(-1.43)	(-0.79)	(-1.25)	(-0.34)	(-0.94)	(-1.88)	(3.06)
$\alpha_u + \alpha_b$: Alpha during bad financial times	0.42	**0.76**	**0.60**	0.06	0.13	0.34	0.27	0.03	0.05	-0.32	**2.00**
	(1.46)	(2.25)	(3.21)	(0.39)	(0.43)	(0.95)	(0.68)	(0.18)	(0.18)	(-0.88)	(5.07)
Drawdowns:											
α_u: Good times alpha	0.19	**0.42**	0.17	**0.24**	**0.58**	**0.59**	**0.71**	0.14	**0.36**	**0.38**	**0.75**
(t)	(1.36)	(2.57)	(1.85)	(3.17)	(3.89)	(3.39)	(3.60)	(1.56)	(2.46)	(2.38)	(3.90)
α_b: Bad times dummy	0.12	0.03	**0.45**	-0.12	-0.19	0.04	0.11	-0.20	-0.19	-0.31	0.59
(t)	(0.39)	(0.09)	(2.27)	(-0.71)	(-0.58)	(0.10)	(0.27)	(-1.07)	(-0.60)	(-0.87)	(1.42)
$\alpha_u + \alpha_b$: Alpha during drawdowns	0.31	0.45	**0.61**	0.13	0.40	**0.63**	**0.82**	-0.06	0.17	0.08	**1.35**
	(1.21)	(1.51)	(3.70)	(0.90)	(1.45)	(1.97)	(2.28)	-(0.40)	(0.64)	(0.26)	(3.83)
Intersection of all bad times measures:											
α_u: Good times alpha	0.17	**0.39**	**0.27**	**0.27**	**0.63**	**0.67**	**0.74**	0.10	**0.39**	**0.43**	**0.75**
(t)	(1.38)	(2.61)	(3.26)	(3.97)	(4.65)	(4.24)	(4.13)	(1.31)	(2.94)	(2.99)	(4.28)
α_b: Bad times dummy	0.37	0.34	0.07	**-0.52**	-0.81	-0.61	-0.02	-0.14	-0.66	**-1.18**	1.26
(t)	(0.94)	(0.74)	(0.27)	(-2.41)	(-1.91)	(-1.24)	(-0.04)	(-0.59)	(-1.58)	(-2.48)	(2.32)
$\alpha_u + \alpha_b$: Alpha during the intersection of all bad times measures	0.54	0.73	0.34	-0.24	-0.17	0.06	0.71	-0.04	-0.26	-0.75	**2.02**
	(1.49)	(1.69)	(1.41)	(-1.21)	(-0.44)	(0.13)	(1.38)	(-0.18)	(-0.68)	(-1.68)	(3.99)